NAVAL POSTGRADUATE SCHOOL
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THESIS

COMPARING THE MAXIMUM LIKELIHOOD METHOD AND A MODIFIED MOMENT METHOD TO FIT A WEIBULL DISTRIBUTION TO AIRCRAFT ENGINE FAILURE TIME DATA

by

Fernando Güimil

September 1997

Principal Advisor: W. Max Woods
Second Reader: Harold J. Larson

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One method used is the Maximum Likelihood Method and assumes that these engine failure times are independent. The other method is a Modified Method of Moments procedure and uses the fact that if time to failure T has a Weibull distribution with scale parameter λ and shape parameter β, then T^{β} has an exponential distribution with scale parameter λ^{β}. The latter method makes no assumption about independent failure times.

A comparison is made from times that are randomly generated with a program. The program generates times in a manner that resembles the way in which engine failures occur in the real world for an engine with three subsystems. These generated operating times between failures for the same engine are not statistically independent. This comparison was extended to real data.

Although the two methods gave good fits, the Maximum Likelihood Method produced a better fit than the Modified Method of Moments. Explanations for this fact are analyzed and presented in the conclusions.

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COMPARING THE MAXIMUM LIKELIHOOD METHOD AND A MODIFIED MOMENT METHOD TO FIT A WEIBULL DISTRIBUTION TO AIRCRAFT ENGINE FAILURE TIME DATA

Fernando Güimil
Lieutenant Commander, Spanish Navy
B.S., Spanish Naval Academy, 1978
Licenciado Ciencias Economicas, Universidad Nacional de Educacion a Distancia, 1993

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of the requirements for the degree of

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September 1997

Author: Fernando Güimil

Fernando Güimil

Approved by: W. Max Woods

W. Max Woods, Thesis Advisor

Harold J. Larson

Harold J. Larson, Second Reader

Richard E. Rosenthal

Richard E. Rosenthal, Chairman
Department of Operations Research

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This thesis provides a comparison of the accuracies of two methods for fitting a Weibull distribution to a set of aircraft engines time-between-failure data.

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THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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EXECUTIVE SUMMARY

This thesis provides a comparison of the accuracies of two methods for fitting a Weibull Distribution to a set of aircraft engines time-between-failure data.

One method used is the Maximum Likelihood Method and assumes that these engine failure times are independent. The other method is a Modified Method of Moments procedure and uses the fact that if time to failure T has a Weibull Distribution with scale parameter λ and shape parameter β, then T^{β} has an exponential distribution with scale parameter λ^{β}. The latter method makes no assumption about independent failure times.

In a fleet of fielded aircraft engines, engines are repaired when a fault is detected. In addition, some critical components are inspected/repaired after a designated number of flight hours. When faults are detected the engine is repaired and the component is replaced on the engine unless the engine has accumulated a total number of flight hours near to the hard inspection time of a critical component(s). If this is the case the critical components(s) are also inspected/repaired. In nearly all cases, the engines are not completely overhauled to restore them to new condition. After repair, components in the engine have mixed ages with respect to flight hours with some components new or repaired. Consequently the times between failures on a specific engine are not independent.

A comparison is made for times that are randomly generated from a program. The program generates times in a manner that resembles the way in which engine failures occur in the real world for an engine with three subsystems. These operating times between failures for the same engine are generated in a way that they are not statistically independent. This is accomplished by using a program in S-Plus which simulates failure times for an engine with three subsystems. The failure time distributions of each one of these subsystems are Weibull with fixed parameters determined a-priori and based on real world experience. The simulation gathers three failure times for each engine. The study includes failures for two different sets of 20 and 100 engines respectively. Once this comparison was completed, the study was extended

to real data using TF-34 engine operating times between failures from the Naval Aviation Logistics Data Analysis (NALDA) data base.

The estimation of the parameters for the Weibull Distribution which fits the failure times was done using spread-sheet software programs. The accuracy of each method was measured using the Kolmogorov-Smirnov test statistic, embedded in the S-Plus software package.

Although the two methods gave good fits, the Maximum Likelihood Method produced a better fit more often than the Modified Method of Moments. Explanations for this fact are analyzed and presented in the conclusions.

LIST OF SYMBOLS, ACRONYMS AND ABBREVIATIONS

β	shape parameter Weibull distribution
λ	reciprocal of scale parameter Weibull distribution
θ	scale parameter Weibull distribution
CDF	Cumulative Distribution Function
ERAP	Engine Reliability Analysis Program
exp	Exponential Distribution
$\|J\|$	Jacobian
K-S	Kolmogorov-Smirnov
MLE	Maximum Likelihood Estimator
MMM	Modified Method of the Moments
n	number of failure times
NALDA	Naval Aviation Logistics Data Analysis
NAWCADPPE	Naval Air Warfare Center Aircraft Division Propulsion and Power Engineering
$\hat{s}$	statistical estimate of the standard deviation
Ti	failure time
$\hat{u}$	statistical estimate of the mean

I. INTRODUCTION

A. BACKGROUND

1. Engine Reliability Analysis Program (ERAP)

The Naval Air Warfare Center Aircraft Division Propulsion and Power Engineering (NAWCADPPE) was tasked with analyzing aircraft engine failure data to estimate aircraft engine reliability and maintainability factors and to assess the effectiveness of existing naval aircraft engine maintenance policies. The ERAP program was established to help accomplish this task. It generates nine reports designed to identify problem areas and trends in engine logistic support, time between engine failures, engine availability, engine readiness degraders, and other engine performance factors. The data base used in the ERAP program is the Naval Aviation Logistics Data Analysis (NALDA) data base. These reports provide summary measures such as mean flight hours between engine failure and other averages and trend lines on these averages. They cannot be used, for example, to compute residual reliabilities of specified additional lifetimes for engines with known number of repairs and operating times since last failure. Nor can they be used to determine optimal maintenance/replacement policies for critical engine components. A more detailed analysis of the engine failure data in the NALDA data base is required to make these types of assessments. Michael R. Caudill [Ref. 1] and John A. Malsbury [Ref. 2] made more detailed analyses of TF-34 engine performance in their Master of Science theses than those provided under the ERAP program. Mark E. Mlikan [Ref. 3] developed an integral equation for determining optimal no-build times for critical components of TF-34 engines relative to their hard inspection times. All three of these theses used a Weibull distribution to model the probability distribution of the flight hours between engine failures for the TF-34 aircraft engines archived in the NALDA data base.

2. NALDA Data Base

NALDA is an operational automated information system. It is a centralized Naval Aviation 3M data base supported by the Defense Megacenter, Mechanicsburg, Pa. A user of the NALDA data base must make coded queries to the system to obtain specific information. This requires considerable experience with NALDA. NALDA contains numerous data elements on Navy aircraft engines by engine type and serial number. Anytime an aircraft engine is inspected, maintained, removed from an aircraft, repaired, transported, reinstalled or discarded, a report is submitted that is translated into the NALDA data base. The NALDA data base must be "cleansed" before it can be used to perform some detailed statistical analyses such as fitting a probability distribution to the empirical cumulative probability distribution of operating times between engine failures. M. Caudill did this cleansing of the NALDA data for TF-34 engines provided to him by support staff at NAWCADPPE. Details of this cleansing process and its purpose are described in Caudill's thesis [Ref. 1].

3. Operating Times Between Engine Failures

The NALDA data base reports the operating time between engine <u>removals</u>. Engines can be removed for non-failure reasons. Caudill [Ref. 1] established a data base of operating time between engine failures for TF-34 engines by engine serial number. These times are "not" statistically independent because of the manner in which aircraft engines are repaired. More often than not, they are not totally overhauled before being reinstalled. Only minor repair is made on some occasions. The operating time between the first and second failure of an engine is dependent on the operating time to first failure and the extent of the repair. Engines with long first failure times and minor first repair will contain many used components with large accumulated operating hours when reinstalled. They will likely have shorter operating times to the next failure than engines with small operating times to first failure. Consequently, the operating times between engine failures, for the same engine, are not statistically independent.

B. FITTING THE PROBABILITY DISTRIBUTION OF OPERATING TIME BETWEEN FAILURES FOR TF-34 ENGINES

1. Weibull Distribution

Aircraft engines wear out. Consequently, any probability distribution used to model their time to failure should reflect wear-out characteristics. The Weibull distribution has these properties. It was used by Caudill [Ref. 1], Malsbury [Ref. 2], and Mlikan [Ref. 3] to model the distribution of operating times between failures for TF-34 engines. They used a modified method of moments procedure to compute the parameters of the fitted Weibull distribution. This method used special properties of the Weibull distribution and the exponential distribution; the sample mean and the sample standard deviation were used to estimate the parameters of the fitted Weibull distribution. It was chosen because it was known to be less sensitive to large positive or negative correlations (dependency) in the data than the more standard methods of maximum likelihood estimation when statistical independence is assumed.

2. Research Question

The analysis performed in this thesis provides a comparison of the accuracies of the maximum likelihood estimation procedure (assuming independence) and the modified method of moments procedure used by Caudill for fitting a Weibull distribution to the probability distribution of operating times between engine failures. These comparisons are made for times that are randomly generated in a manner that resembles the way in which engine failure times occur in the real world. These randomly generated operating times between failures for the same engine will not be statistically independent. Once this comparison was completed, it was extended to real data using TF-34 engine operating times between failures from the NALDA data base. [Ref. 1]

II. GENERATION OF FAILURE TIMES

A. PROCEDURE

To generate the failure times, a program in S-Plus (See Appendix A) was developed with the following properties.

1. Each engine has 3 components (subsystems).
2. The failure time distributions of each of these components are Weibull distributions with fixed parameters determined a-priori and based on experience.
3. The data generated is a Type II Censored Data Set, and a total of 3 failure times are gathered for each engine.

With these criteria, the program generates the failure times in the following way:

Step 1: Three Weibull random numbers are generated. They represent the first failure time that each one of the three components could have. These values constitute the vector *faults1*. Three additional Weibull random numbers are generated. They represent the second failure time that each one of the three components could have. These values constitute the vector *faults2*.Three additional Weibull random numbers are generated. They represent the third failure time that each one of the three components could have. These values constitute the vector *faults3*.

Step 2: The minimum value of the vector *faults1* is selected, this value is the time of the first failure of the engine. The program creates a new vector *newfail2* that is the difference between the vector *faults1* and the minimum value selected; it is clear that this new vector has a component that is 0. This component is replaced in the vector *newfail2* by the element of the vector *faults2* that has the same index as the first failure.

Step 3: At this point the engine has two components that have accumulated working hours and one that has been just repaired. The vector *newfail2* has two elements that have the remainder of the theoretical first failure for these components and one element that is the theoretical second failure for the other component. So, again, the minimum element of this vector is selected and it is the second failure time of the engine.

Step 4: The program creates a new vector called *newfail3* that is the difference between the vector *newfail2* and this minimum value selected; it is clear that this vector has a component that is 0.

If the second engine failure time has taken place in the same component that had the first failure, the 0 element is replaced by the element of the vector *faults3* whose index matches that of the component that has had the two first failures or by the element of the vector *faults2* whose index matches with the component that had the second failure. Again, there is an element in the vector *faults3* that is a theoretical failure time for one component just repaired, and two elements that have the remainder of theoretical failures. So, once again the minimum element of this vector will be the value of the third engine failure time. The vector *enginefails* gives the three random failure times of the engine.

B. GENERATION

An S-Plus program was developed to generate, in the bank of engines, the three failure times for the engine. The program can be run for different numbers of engines. In this thesis the program was run for 100 engines with a total of 300 failure times and for 20 engines with 60 failure times. This was done to determine if the number of engines has an apparent effect in the results of the study. For each one of these engine sets, 30 replications were generated. These replications were exported from S-Plus to Excel with the export tool via the S-Plus file-Export menu and used to assess averages on the accuracy of the two methods of estimation.

The first replication for both 20 and 100 engines can be seen in the Appendix A, all of the other replications have been retained on diskettes attached to the master copy of this thesis.

III. ESTIMATION OF PARAMETERS WITH THE MAXIMUM LIKELIHOOD AND THE MODIFIED MOMENT METHOD

A. PROCEDURE

The two parameter Weibull distribution analyzed in this thesis has a probability density function given by:

$$f_T(t) = \beta\lambda^{\beta} t^{\beta-1} e^{-(\lambda t)^{\beta}}, \tag{1}$$

or equivalently

$$f_T(t) = \beta\theta^{-\beta} t^{\beta-1} e^{-(\frac{t}{\theta})^{\beta}}, \tag{2}$$

where $\theta = \frac{1}{\lambda}$. Customarily β is called the shape parameter and λ (or θ) is called the scale parameter.

To estimate the parameters, the capabilities of the spread sheet program Excel were used. The failure times are generated in S-Plus and exported to Excel, where they are displayed automatically on the Sheet1 Worksheet of Excel.

The Maximum Likelihood estimators are calculated based on well known formulas [Ref. 4]. Solve numerically for $\hat{\beta}$:

$$\frac{n}{\beta} + \sum_{i=1}^{n} LnTi - \frac{n\sum_{i=1}^{n} Ti^{\beta} LnTi}{\sum_{i=1}^{n} Ti^{\beta}} = 0 \,. \tag{3}$$

Then:

$$\hat{\lambda} = \left(\frac{n}{\sum_{i=1}^{n} Ti^{\hat{\beta}}} \right)^{\frac{1}{\hat{\beta}}}, \qquad \textbf{(4)}$$

$$\hat{\theta} = \frac{1}{\hat{\lambda}}. \qquad \textbf{(5)}$$

The parameters for the Modified Method of Moments were calculated based on the fact that if T has a Weibull distribution then T^{β} has a Exponential distribution with a scale parameter of λ^{β}. The following demonstration is from [Ref 5]:

Considering the Weibull distribution given by:

$$f_t(t) = \beta\lambda^{\beta} t^{\beta-1} e^{-(\lambda t)^{\beta}}. \qquad \textbf{(6)}$$

Let $x = t^{\beta}$ so $t = x^{1/\beta}$,

$$\frac{dt}{dx} = \frac{1}{\beta} x^{\frac{1}{\beta}-1}. \qquad \textbf{(7)}$$

Then

$$\begin{aligned} f_X(x) &= |J| f(t(x)) = \left| \frac{1}{\beta} x^{\frac{1}{\beta}-1} \right| f_t(x^{\frac{1}{\beta}}) \\ &= \frac{1}{\beta} x^{\frac{1}{\beta}-1} \beta\lambda^{\beta} x^{\frac{\beta-1}{\beta}} e^{-\lambda^{\beta} x} \\ &= \lambda^{\beta} e^{-\lambda^{\beta} x} \end{aligned} \qquad \textbf{(8)}$$

Therefore:

$$X \sim \exp(\lambda^{\beta}). \tag{9}$$

It is well known that the mean and standard deviation of the exponential distribution are equal. Therefore their statistical estimates $\hat{u}$ and $\hat{s}$ should be nearly equal. That is

$$\hat{s} = \left[\frac{1}{n-1}\sum_{i=1}^{n}(T_i^{\beta} - \overline{(T_i^{\beta})})^2\right]^{\frac{1}{2}} \approx \frac{1}{n}\sum_{i=1}^{n} T_i^{\beta} = \hat{u}. \tag{10}$$

where the T_i are the times between engine failures.

The solution for β in equation (10) will be denoted by $\bar{\beta}$.

Since $E(T_i^{\beta}) = \theta^{\beta} = \left(\frac{1}{\lambda}\right)^{\beta}$, the sample mean, $\frac{1}{n}\sum_{1}^{n} T_i^{\beta}$ estimates θ^{β}, therefore:

$$\bar{\theta} = \left[\frac{1}{n}\sum_{i=1}^{n} T_i^{\bar{\beta}}\right]^{\frac{1}{\bar{\beta}}}, \tag{11}$$

This is the same estimate obtained for the MLE of θ.

B. ESTIMATION

To carry out the estimation of parameters with the two methods, two Macros in Visual Basic were developed. One Macro for 20 engines (See Appendix B), that uses 60 failure times, and the other for 100 engines (See Appendix C), that uses 300 failure times.

Basically, these two Macros are similar and therefore they, and procedures for using them, can be similarly described. Once the data is exported to Excel, it appears on *Sheet1.* At this point it is necessary to open the file with the Macro, called *Macro20.xls* for the sample of 20 engines and *Macro100.xls* for the sample of 100 engines respectively. Next the window with the Macro is hidden from the same *Sheet1.*Then the Macro can be run. To run the Macro,

choose Macro from the Tools menu. From Macro/Name Reference choose the name of the Macro and click on run. The Macro does all the previous calculations and operations needed to estimate the parameters and to entitle the entire Worksheet. After these calculations the Macro stops. At this point it is necessary to introduce the following data:

In Cell B6 introduce the value previously calculated by the macro for the initial value of the shape parameter which is in Cell B4.

This initial parameter was introduced to assure the convergence with the Tool Goal Seeker, because without it, a solution may not be reached. The initial value is given by [Ref. 6]:

$$\beta_0 = \left\{ \frac{\frac{6}{\pi^2}\left[\sum_{i=1}^{n}(LnTi)^2 - (\sum_{i=1}^{n} LnTi)^2 / n\right]}{n-1} \right\}^{-\frac{1}{2}} . \tag{12}$$

Using these choices for the initial shape parameter, an average of only 3.5 Newton iterations were needed to achieve four-place accuracy in the solution for $\bar{\beta}$.

After introducing the initial value given in Cell B4 into Cell B6, cell C6 must be highlighted. Then the Tools Menu is selected, Goal Seek is selected, a 0 is placed in the "To Value," and B6 is entered as the "Changing Cell."

Equation (3) is in Cell C6 and the Goal Seeker searches for the value of β which satisfies that equation. There were no non-convergence problems in any of the 70 replications made with that formula. Once the solution is achieved, the final value for the shape parameter appears in Cell B6. The solutions for λ and θ appear in Cell B9 (Equation (4)) and B11 (Equation (5)) respectively. These solutions for β, λ and θ are the maximum likelihood estimates for these parameters of the Weibull distribution that is used to model the probability distribution of the family of times between failures on all engines. These values are printed on the left side of Sheet2.

On the right side of Sheet 2 are the calculations carried out by the Modified Moment Method.

There were convergence problems in calculating β using Equation (10). Amazingly the same formula may converge or not depending on the terms on each side. Through trial and error it was discovered that by writing the formula in the following way it was possible to get convergence in all replications:

$$\frac{n}{\sqrt{n-1}} = \frac{\sum_{i=1}^{n} Ti^{\beta}}{\left[\sum_{i=1}^{n} (Ti^{\beta} - \overline{(Ti^{\beta})})^2\right]^{0.5}} . \tag{13}$$

In Cell Q6 it is necessary to introduce the initial shape parameter given in Cell Q4 (Equation (12)). Once done, highlight Cell P7 (Equation (13)), select the Tools Menu, select Goal Seek and depending on the number of engines, enter in "To Value" 7.811334 (20 engines, 60 failure times, $60/\sqrt{59}$) or 17.349447 (100 engines, 300 failure times, $300/\sqrt{299}$). In the "Changing Cell, Q6," the Goal Seeker looks for the β value that will satisfy Equation (13). Cells Q9 and Q11 contain the values for the scale parameter λ and θ determined by Equation (11) and (5).

A total of 30 replications for each set of engines were done. Afterwards, for the reasons explained later, 10 more replications were done using real data.

Printouts of the failure times and parameter calculations for the 20 and 100 engines first replication are provided in Appendix E.

A summary of the parameters obtained is shown in the following tables:

Table 1. Twenty Engines = 60 Failure Times

20 Engines = 60 Failure Times

Replication Number	Maximum Likelihood Estimator Shape	Maximum Likelihood Estimator Scale	Modified Moment Method Shape	Modified Moment Method Scale
1	0.973000000	251.705300000	0.985725000	253.139700000
2	1.105104390	279.481900000	1.129925000	281.796500000
3	1.052707000	242.343600000	1.036553000	240.792500000
4	1.005354000	282.577600000	1.079900000	291.030800000
5	1.211857000	272.413900000	1.102670000	263.279100000
6	1.523761000	277.443800000	1.442500000	273.090800000
7	1.053815000	243.377800000	1.309674000	262.946000000
8	1.015065000	330.527000000	1.006751000	329.425200000
9	0.961137000	251.822700000	1.078819000	264.611000000
10	1.108272000	263.760200000	1.084296000	261.569300000
11	1.258861000	317.378200000	1.395249000	328.224900000
12	1.358459000	245.953800000	1.211490000	237.176300000
13	1.323305000	281.038800000	1.461631834	289.757400000
14	1.054978965	223.276800000	1.249979811	236.965600000
15	1.228101693	264.364500000	1.182801000	260.845800000
16	1.161814103	243.734600000	1.239576000	249.537300000
17	1.355617000	268.067900000	1.367488000	268.798900000
18	1.308393000	309.037200000	1.357252000	312.609300000
19	1.391697000	284.440700000	1.406588000	285.359900000
20	1.141031467	242.289400000	1.324936416	255.330700000
21	1.336786000	271.132800000	1.223524000	263.236700000
22	1.389460000	244.898600000	1.280865198	238.618200000
23	1.200747000	288.213200000	1.263844000	293.349300000
24	1.297427000	293.585500000	1.260780021	290.891900000
25	1.400818000	310.907900000	1.325062000	305.455500000
26	1.231510000	277.128900000	1.320218000	283.439100000
27	1.048038281	277.331100000	1.043436000	276.830600000
28	1.093433000	273.733700000	1.248177000	286.965800000
29	1.295133208	246.789500000	1.240951783	243.300600000
30	1.224688831	303.055100000	1.402646081	317.067700000
Average	1.203679065	272.060400000	1.235443638	274.848080000
Std. Dev.	0.145886716	25.376732459	0.133749260	25.937151577

Table 2. One-Hundred Engines = 300 Failure Times

100 Engines = 300 Failure Times

Replication Number	Maximum Likelihood Estimator Shape	Maximum Likelihood Estimator Scale	Modified Moment Method Shape	Modified Moment Method Scale
1	1.261028000	292.556900000	1.288232000	294.659500000
2	1.147177000	258.382300000	1.182793000	261.246400000
3	1.192249000	267.935700000	1.250620000	272.423600000
4	1.191393000	301.254500000	1.243339000	305.757700000
5	1.100167000	265.867700000	1.187671000	272.564100000
6	1.132621000	273.303500000	1.205131000	279.422100000
7	1.220806000	282.032200000	1.334481000	290.504100000
8	1.206135000	267.017000000	1.246956000	270.084700000
9	1.254734000	291.692500000	1.271828000	293.041800000
10	1.284450000	285.463200000	1.292318000	286.039200000
11	1.277785000	285.850800000	1.387920000	293.508300000
12	1.274025000	263.121900000	1.341512000	267.591700000
13	1.192249000	267.935700000	1.250620000	272.423600000
14	1.194163000	271.996400000	1.228539000	274.677200000
15	1.216995000	272.494100000	1.231497000	273.612100000
16	1.169718000	293.370600000	1.200586000	296.122500000
17	1.283386000	255.640200000	1.259945000	254.068200000
18	1.416971000	294.799600000	1.422087000	295.112700000
19	1.264662000	268.682800000	1.379585000	276.306900000
20	1.342221000	279.712400000	1.448534000	286.372700000
21	1.229806000	287.109500000	1.269967000	290.254800000
22	1.199120000	267.635600000	1.281756000	273.804000000
23	1.189326000	288.996500000	1.207016000	290.494900000
24	1.199541000	263.550100000	1.346121000	273.747200000
25	1.268509000	261.778100000	1.201983000	256.990000000
26	1.257310000	256.670600000	1.249969000	256.167900000
27	1.274659000	270.850700000	1.311836000	273.423600000
28	1.208075000	276.652800000	1.237968000	278.973700000
29	1.220756000	264.013900000	1.219361000	263.912300000
30	1.127942000	279.203900000	1.173691000	283.295300000
Average	1.226599300	275.185723333	1.271795400	278.553426667
Std. Dev.	0.063780678	12.379509005	0.070516179	13.009908323

IV. ACCURACY ANALYSIS

A. PROCEDURE

The accuracy of each method was measured using the Kolmogorov-Smirnov goodness of fit Test and the S-Plus software package. The null hypothesis in S-Plus is "the True cdf (empirical) equals the Weibull distribution". This test has the advantage that it uses continuous populations. Therefore it is not necessary to group the data.

This test has the disadvantage that when distribution parameters are estimated from the same sample of data, the test produces a conservative result; that is, the probability of a Type I error will be smaller than specified [Ref 6]. This drawback has only a relative importance in the present study. The K-S Test p-value is a conservative result whether the parameters are estimated by the Maximum Likelihood Method or the Modified Moment Method.

A S-Plus function called *Tocompare* (See Appendix E) was created to calculate the K-S statistics for the set of parameters calculated in each replication using both methods. This S-Plus function computes the K-S Test Statistic (*Supremum*) and the p-value for each method, and it also plots the empirical and hypothesized distributions. In Appendix E the reader will find the output of the function for the first replication of each set of parameters.

B. ACCURACY

The calculations for all replications and the values obtained for each set of engines are presented in Tables 3 and 4. The letter " B" marks the method that achieved a better fit, using as reference the K-S statistic; that is, the method with the smaller K-S statistic is the more accurate procedure for that replication.

For the set of 20 engines, the MLE method gave a better fit in 17 of the 30 replications. For the set of 100 engines, the MLE method gave a better fit in 20 of the 30 replications.

Table 3. Better Fit Twenty Engines

20 Engines = 60 Failures times

Replication Number	M.L.E. shape	M.L.E. scale	M.L.E. k-s	M.L.E. p-value		M.M.M. shape	M.M.M. scale	M.M.M. k-s	M.M.M. p-value	
1	0.973000000	251.705300000	0.054600000	0.994000000		0.985725000	253.139700000	0.050900000	0.997700000	B
2	1.105104390	279.481900000	0.088300000	0.737300000		1.129925000	281.796500000	0.081000000	0.826600000	B
3	1.052707000	242.343600000	0.076300000	0.876000000		1.036553000	240.792500000	0.073000000	0.906600000	B
4	1.005354000	282.577600000	0.061700000	0.976300000	B	1.079900000	291.030800000	0.086100000	0.765600000	
5	1.211857000	272.413900000	0.101800000	0.562800000		1.102670000	263.279100000	0.097300000	0.621600000	B
6	1.523761000	277.443800000	0.086700000	0.757300000		1.442500000	273.090800000	0.081000000	0.826400000	B
7	1.053815000	243.377800000	0.091500000	0.696500000	B	1.309674000	262.946000000	0.115700000	0.398400000	
8	1.015065000	330.527000000	0.052700000	0.996200000	B	1.006751000	329.425200000	0.053100000	0.995900000	
9	0.961137000	251.822700000	0.088000000	0.742000000	B	1.078819000	264.611000000	0.098600000	0.604700000	
10	1.108272000	263.760200000	0.064500000	0.964400000		1.084296000	261.569300000	0.061500000	0.977100000	B
11	1.258861000	317.378200000	0.101500000	0.566300000	B	1.395249000	328.224900000	0.126600000	0.290900000	
12	1.358459000	245.953800000	0.104300000	0.531000000	B	1.211490000	237.176300000	0.121500000	0.338000000	
13	1.323305000	281.038800000	0.075700000	0.881800000	B	1.461631834	289.757400000	0.110800000	0.452600000	
14	1.054978965	223.276800000	0.088000000	0.741900000		1.249979811	236.965600000	0.087200000	0.751700000	B
15	1.228101693	264.364500000	0.070300000	0.928000000		1.182801000	260.845800000	0.065700000	0.958200000	B
16	1.161814103	243.734600000	0.078800000	0.850400000	B	1.239576000	249.537300000	0.101500000	0.567200000	
17	1.355617000	268.067900000	0.070200000	0.929200000	B	1.367488000	268.798900000	0.072500000	0.910600000	
18	1.308393000	309.037200000	0.049400000	0.998600000	B	1.357252000	312.609300000	0.054400000	0.994300000	
19	1.391697000	284.440700000	0.083700000	0.794200000		1.406588000	285.359900000	0.083400000	0.798500000	B
20	1.141031467	242.289400000	0.091800000	0.692600000	B	1.324936416	255.330700000	0.116800000	0.386000000	
21	1.336786000	271.132800000	0.107000000	0.498400000		1.223524000	263.236700000	0.077500000	0.863800000	B
22	1.389460000	244.898600000	0.085000000	0.778500000	B	1.280865198	238.618200000	0.092700000	0.681600000	
23	1.200747000	288.213200000	0.083700000	0.795100000	B	1.263844000	293.349300000	0.096600000	0.629800000	
24	1.297427000	293.585500000	0.058400000	0.986700000	B	1.260780021	290.891900000	0.059800000	0.982700000	
25	1.400818000	310.907900000	0.071100000	0.921800000		1.325062000	305.455500000	0.071100000	0.922100000	B
26	1.231510000	277.128900000	0.062500000	0.973200000		1.320218000	283.439100000	0.057400000	0.989000000	B
27	1.048038281	277.331100000	0.078500000	0.853300000		1.043436000	276.830600000	0.077100000	0.867800000	B
28	1.093433000	273.733700000	0.078000000	0.859200000	B	1.248177000	286.965800000	0.084100000	0.790300000	
29	1.295133208	246.789500000	0.055600000	0.992500000	B	1.240951783	243.300600000	0.067600000	0.947000000	
30	1.224688831	303.055100000	0.083900000	0.792200000	B	1.402646081	317.067700000	0.095900000	0.639300000	
	mean	mean	mean	mean		mean	mean	mean	mean	
	1.203679065	272.060400000	0.078116667	0.822256667	17	1.235443638	274.848080000	0.083946667	0.756066667	13
	stdev	stdev	stdev	stdev		stdev	stdev	stdev	stdev	
	0.145886716	25.376732459	0.015610639	0.146004782		0.133749260	25.937151577	0.020788358	0.214131410	

Table 4. Better Fit One-Hundred Engines

100 Engines = 300 Failures times

Replication Number	M.L.E. shape	M.L.E. scale	M.L.E. k-s	M.L.E. p-value		M.M.M. shape	M.M.M. scale	M.M.M. k-s	M.M.M. p-value	
1	1.261028000	292.556900000	0.031900000	0.920600000	B	1.288232000	294.659500000	0.038200000	0.774000000	
2	1.147177000	258.382300000	0.027400000	0.977600000	B	1.182793000	261.246400000	0.032700000	0.905100000	
3	1.192249000	267.935700000	0.047000000	0.522300000	B	1.250620000	272.423600000	0.051000000	0.416900000	
4	1.191393000	301.254500000	0.055800000	0.306900000		1.243339000	305.757700000	0.046600000	0.532800000	B
5	1.100167000	265.867700000	0.060600000	0.220000000		1.187671000	272.564100000	0.045600000	0.561900000	B
6	1.132621000	273.303500000	0.043200000	0.630900000		1.205131000	279.422100000	0.042500000	0.651400000	B
7	1.220806000	282.032200000	0.034300000	0.873100000	B	1.334481000	290.504100000	0.048900000	0.469300000	
8	1.206135000	267.017000000	0.037900000	0.782600000		1.246956000	270.084700000	0.036300000	0.823000000	B
9	1.254734000	291.692500000	0.021600000	0.999000000	B	1.271828000	293.041800000	0.025500000	0.989900000	
10	1.284450000	285.463200000	0.022700000	0.997900000		1.292318000	286.039200000	0.020800000	0.999500000	B
11	1.277785000	285.850800000	0.044600000	0.589000000	B	1.387920000	293.508300000	0.058300000	0.260200000	
12	1.274025000	263.121900000	0.047400000	0.511700000	B	1.341512000	267.591700000	0.048900000	0.469900000	
13	1.192249000	267.935700000	0.029400000	0.957700000	B	1.250620000	272.423600000	0.035300000	0.848500000	
14	1.194163000	271.996400000	0.039600000	0.735300000	B	1.228539000	274.677200000	0.048600000	0.477100000	
15	1.216995000	272.494100000	0.031400000	0.928900000		1.231497000	273.612100000	0.029400000	0.957500000	B
16	1.169718000	293.370600000	0.034800000	0.861700000	B	1.200586000	296.122500000	0.041300000	0.685300000	
17	1.283386000	255.640200000	0.031800000	0.922000000	B	1.259945000	254.068200000	0.033000000	0.898900000	
18	1.416971000	294.799600000	0.039800000	0.729300000		1.422087000	295.112700000	0.038700000	0.759500000	B
19	1.264662000	268.682800000	0.053800000	0.349300000	B	1.379585000	276.306900000	0.065700000	0.150500000	
20	1.342221000	279.712400000	0.039700000	0.730800000	B	1.448534000	286.372700000	0.067500000	0.129900000	
21	1.229806000	287.109500000	0.041300000	0.685900000		1.269967000	290.254800000	0.040300000	0.713200000	B
22	1.199120000	267.635600000	0.036000000	0.830800000	B	1.281756000	273.804000000	0.044400000	0.596300000	
23	1.189326000	288.996500000	0.032800000	0.903200000	B	1.207016000	290.494900000	0.033600000	0.886800000	
24	1.199541000	263.550100000	0.032000000	0.919200000	B	1.346121000	273.747200000	0.048900000	0.469200000	
25	1.268509000	261.778100000	0.028200000	0.971000000	B	1.201983000	256.990000000	0.041500000	0.680900000	
26	1.257310000	256.670600000	0.026400000	0.984800000	B	1.249969000	256.167900000	0.027700000	0.975800000	
27	1.274659000	270.850700000	0.038700000	0.759500000	B	1.311836000	273.423600000	0.048400000	0.482200000	
28	1.208075000	276.652800000	0.035200000	0.851300000		1.237968000	278.973700000	0.028200000	0.971300000	B
29	1.220756000	264.013900000	0.033800000	0.882800000	B	1.219361000	263.912300000	0.034200000	0.874100000	
30	1.127942000	279.203900000	0.035700000	0.838800000		1.173691000	283.295300000	0.031400000	0.928300000	B
	mean	mean	mean	mean		mean	mean	mean	mean	
	1.226599300	275.185723333	0.037160000	0.772463333	20	1.271795400	278.553426667	0.041113333	0.677973333	10
	standev	standev	standev	standev		standev	standev	standev	standev	
	0.063780678	12.379509005	0.009086679	0.209187169		0.070516179	13.009908323	0.010947594	0.247541499	

If the MLE and MMM methods are equally good then one would expect that the probability the MLE gives a better fit (using the K-S statistic) should be 0.5. Let π represent the probability the MLE gives a better fit; the above sets of data can then be used to test $H_0 : \pi = 0.5$ versus $H_a : \pi \neq 0.5$. For the set of 20 engines, 17 successes were observed out of 30 replications (MLE gave a smaller K-S value than MMM); the p-value for testing $H_0 : \pi = 0.5$ is 0.5847 so H_0 would be accepted for any $\alpha < 0.5847$. For the set of 100 engines, 20 of the 30 replications gave successes; in this case the p-value is 0.0987 and H_0 is rejected for any $\alpha \geq 0.0987$. Certainly this comparison of MLE and MMM, using the binomial test, shows no real difference for 20 engines and marginally indicates superiority of MLE for 100 engines.

If the K-S statistic mean and standard deviation are analyzed it is clear that the MLE. gives the smaller mean and standard deviation of the two cases.

For this data, it seems that despite the fact that there is some dependence between the failure times, the MMM does not give a better fit than the MLE. This is true at least for the failure times generated with the simulation program used in this study. This is likely due to the fact that the failure times for different engines (among engines) are independent.

At this point real failure data was examined. This real data is from [Ref. 1], and from the cleansed data collected in Appendix B (Five-Year Time Between Failures Data Base). This data was cleansed, removing zeros as failure times. Ten samples of data were taken from the data set of operating times between failures on TF 34 aircraft engines. Five samples were of 60 failures times and five samples were of 300 failure times.

The same procedure to estimate the parameters and analyze the accuracy of the results were applied. The results are shown in Tables 5 and 6.

With the real data the result is the same, a better fit is obtained more often with the MLE method than with the MMM method. In the 60 failure times case the MLE method was the better procedure in four of the five samples, and in the 300 failure times case it was better three of the five samples using the K-S statistic as measure of accuracy.

It is important to point out that in these real data, the engine structure is unknown. There is no information about how many elements the engine has, or of course the way that these components failed.

Table 5. Estimate Parameters and Analyze Accuracy Twenty Engines

20 Engines = 60 Failures times

Replication Number	M.L.E. shape	M.L.E. scale	M.L.E. k-s	M.L.E. p-value		M.M.M. shape	M.M.M. scale	M.M.M. k-s	M.M.M. p-value	
1	1.554927	651.243400	0.103000	0.547400	B	1.732451	669.495500	0.103200	0.544600	
2	1.435165	710.748800	0.079700	0.840000	B	1.735014	748.435600	0.130200	0.261300	
3	1.531311	603.432400	0.072600	0.909800		1.635563	613.677100	0.051800	0.997100	B
4	1.436419	609.475100	0.103800	0.537100	B	1.485100	615.440400	0.114900	0.406200	
5	1.535733	579.331200	0.135400	0.221300	B	1.490634	574.988000	0.145900	0.155400	
	mean	**mean**	**mean**	**mean**		**mean**	**mean**	**mean**	**mean**	
	1.414134	642.431620	0.048360	0.511380	4	1.587754	663.677300	0.057660	0.289300	1
	stdev	**stdev**	**stdev**	**stdev**		**stdev**	**stdev**	**stdev**	**stdev**	
	0.045711	14.890226	0.008406	0.185231		0.023623	16.834529	0.006280	0.138620	

C. ANALYSIS OF THE RESULTS

The author expected " a-priori" a better fit with the MMM than with the MLE, based on the fact that the generated operating times between failures for the same engine are not statistically independent. But the resulting fit for the data was better with the MLE more often than with the MMM. This result is further analyzed here, by examining how the dependence between failure times affect each method.

To assess dependence between failure times, the correlation and the grade of dependence between the failure times was calculated for each replication. The correlation coefficients were calculated and added at the end of each replication (See Appendix A). In addition the Kendall-Tau independence test was performed. The null hypothesis in S-Plus for this test is " Tau is equal to 0" that is the two samples are independent. This analysis was performed for all possible pairwise combinations of the failure times of an engine; namely first-second, first-third, second-third correlations, tau coefficients and p-values were calculated. For the Kendall-Tau test the minimum, maximum and mean p-values of those pairwise

combinations of each replication were added to the tables. The results are shown in Tables 7 through 10.

Table 6. Estimate Parameters and Analyze Accuracy One-Hundred Engines

100 Engines = 300 Failures times

Replication Number	M.L.E. shape	M.L.E. scale	M.L.E. k-s	M.L.E. p-value		M.M.M. shape	M.M.M. scale	M.M.M. k-s	M.M.M. p-value	
1	1.462681	618.916600	0.047200	0.515100		1.579829	632.033000	0.045700	0.559000	B
2	1.439461	663.159800	0.045200	0.571700	B	1.586168	681.405000	0.063000	0.184300	
3	1.366409	641.062700	0.040500	0.709200	B	1.564422	666.063600	0.057300	0.279000	
4	1.351920	636.911100	0.064600	0.163100		1.575530	665.621100	0.061900	0.200100	B
5	1.450201	652.107900	0.044300	0.597800	B	1.632821	673.263800	0.060400	0.224100	
	mean	**mean**	**mean**	**mean**		**mean**	**mean**	**mean**	**mean**	
	1.414134	642.431620	0.048360	0.511380	3	1.587754	663.677300	0.057660	0.289300	2
	stdev	**stdev**	**stdev**	**stdev**		**stdev**	**stdev**	**stdev**	**stdev**	
	0.045711	14.890226	0.008406	0.185231		0.023623	16.834529	0.006280	0.138620	

From those tables may be deduced that with the set of 20 engines the correlation is clearly larger than with 100 engines. When the independence is tested with the Kendall-Tau test, the values of the tau's coefficients are similar to the correlation coefficients, and have larger values for the set of 20 engines than the set of 100 engines. In general for this set of data, the correlation coefficients are small and the p-values in the Kendall-Tau are not significant enough to reject the null hypothesis of independence at the usual values. That means the dependence obtained in the simulation is small, specially with the set of 100 engines; this may be unfavorable to the performance of the MMM.

To summarize the results, Figures 1 through 4 present the min and max Kendall-Tau test p-value for each replication for the method that gives the best fit.

From these figures and specially from the max p-values, it may be deduced that, for this data, in general the larger the max p-value (more independence) the better the fit with the MLE method. Conversely the smaller the max p-value (less independence) the better the fit with the MMM method.

Table 7. Correlation - 20 Engines

Correlation Coefficients for 20 engines

rep1		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.099201099	1	
	Failure3	-0.325682845	0.177625785	1
rep2		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.251973035	1	
	Failure3	0.460278255	-0.055862733	1
rep3		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.292364468	1	
	Failure3	-0.171745200	-0.161919234	1
rep4		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.266964793	1	
	Failure3	0.375627368	-0.020033442	1
rep5		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.125786346	1	
	Failure3	-0.240386929	-0.146477812	1
rep6		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.049045517	1	
	Failure3	0.141463053	0.466711164	1
rep7		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.334293594	1	
	Failure3	-0.102335000	0.108178021	1
rep8		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.362844249	1	
	Failure3	-0.153316474	-0.014631447	1
rep9		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	0.138519930	1	
	Failure3	-0.383549222	0.096601004	1
rep10		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.205353856	1	
	Failure3	-0.202946234	-0.062062633	1
rep11		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.23980229	1	
	Failure3	0.164227499	-0.089182145	1
rep12		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.291025767	1	
	Failure3	-0.283057458	-0.216673298	1
rep13		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	0.010730000	1	
	Failure3	-0.148510000	0.018339000	1
rep14		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.431010000	1	
	Failure3	-0.284860000	-0.144210000	1
rep15		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.075920000	1	
	Failure3	0.111095000	-0.381490000	1

rep16		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	0.381725000	1	
	Failure3	0.095575000	-0.033370000	1
rep17		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.214350000	1	
	Failure3	-0.567440000	0.066732000	1
rep18		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.019240000	1	
	Failure3	0.089556000	0.366561000	1
rep19		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	0.062676000	1	
	Failure3	-0.211740000	-0.000530000	1
rep20		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.227550000	1	
	Failure3	0.225389000	-0.371290000	1
rep21		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.267500000	1	
	Failure3	0.140151000	-0.088550000	1
rep22		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.237640000	1	
	Failure3	0.051596000	-0.117030000	1
rep23		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.300890000	1	
	Failure3	0.077012000	-0.043760000	1
rep24		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.159170000	1	
	Failure3	0.008203000	0.087306000	1
rep25		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	0.368302000	1	
	Failure3	-0.205760000	-0.176000000	1
rep26		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.076760000	1	
	Failure3	0.025953000	-0.17618	1
rep27		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.059610000	1	
	Failure3	0.009859000	0.421248000	1
rep28		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.166470000	1	
	Failure3	-0.408460000	-0.182890000	1
rep29		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	-0.183030000	1	
	Failure3	-0.007070000	-0.235020000	1
rep30		Failure1	Failure2	Failure3
	Failure1	1		
	Failure2	0.011941000	1	
	Failure3	0.144591000	0.137677000	1

Table 8. Correlation - 100 Engines

Correlation Coefficients for 100 engines

rep1	Failure1	Failure2	Failure3
Failure1	1		
Failure2	0.012345873	1	
Failure3	-0.151985402	0.027275574	1
rep2	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.053286726	1	
Failure3	-0.155893588	-0.044274299	1
rep3	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.250749723	1	
Failure3	0.062100274	-0.094868699	1
rep4	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.218109532	1	
Failure3	-0.144770967	-0.109527904	1
rep5	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.150008998	1	
Failure3	0.132158154	-0.078813135	1
rep6	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.079147073	1	
Failure3	-0.240225116	-0.170526078	1
rep7	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.065957419	1	
Failure3	-0.065309916	-0.149967098	1
rep8	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.087122634	1	
Failure3	0.009267376	-0.198193315	1
rep9	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.024708152	1	
Failure3	-0.088238686	-0.153996216	1
rep10	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.085599667	1	
Failure3	-0.208242972	-0.04475226	1
rep11	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.030401682	1	
Failure3	0.019141802	0.012598681	1
rep12	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.243204236	1	
Failure3	0.016811427	-0.06918138	1
rep13	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.250749723	1	
Failure3	0.062100274	-0.094868699	1
rep14	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.064039892	1	
Failure3	-0.077445912	-0.077941408	1
rep15	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.098178961	1	
Failure3	0.077241660	-0.170746677	1
rep16	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.343673022	1	
Failure3	-0.065777779	-0.047403929	1
rep17	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.095841896	1	
Failure3	0.153445659	-0.210881943	1
rep18	Failure1	Failure2	Failure3
Failure1	1		
Failure2	0.052473163	1	
Failure3	0.041985512	0.022318537	1
rep19	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.062164561	1	
Failure3	-0.133619298	-0.160869201	1
rep20	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.083449627	1	
Failure3	-0.003128899	0.055670639	1
rep21	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.110695458	1	
Failure3	-0.170345154	-0.103486186	1
rep22	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.180701469	1	
Failure3	0.156143924	-0.219732137	1
rep23	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.073156230	1	
Failure3	-0.171758743	-0.083586000	1
rep24	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.063590173	1	
Failure3	-0.106654719	-0.162453112	1
rep25	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.084052612	1	
Failure3	0.049055891	-0.063152872	1
rep26	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.066404142	1	
Failure3	0.099904638	-0.219739207	1
rep27	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.056026419	1	
Failure3	-0.077399648	-0.002855117	1
rep28	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.081782938	1	
Failure3	-0.061450025	0.020642837	1
rep29	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.243170112	1	
Failure3	0.093441256	-0.10701961	1
rep30	Failure1	Failure2	Failure3
Failure1	1		
Failure2	-0.23939123	1	
Failure3	-0.020603539	-0.257213197	1

Table 9. Kendall -Tau Test - 20 Engines

KENDALL-TAU TEST OF INDEPENDENCE FOR THE SET OF 20 ENGINES

Replication Number	tau12	pvalue	tau13	pvalue	tau23	pvalue	mean	min	max	best method
1	-0.084	0.604	-0.242	0.136	0.126	0.436	0.392	0.136	0.604	m.m.m
2	-0.211	0.194	0.221	0.173	-0.062	0.697	0.355	0.173	0.697	m.m.m
3	-0.116	0.475	-0.158	0.330	-0.095	0.559	0.455	0.330	0.559	m.m.m
4	-0.179	0.270	0.105	0.516	0.126	0.436	0.408	0.270	0.516	m.l.e.
5	-0.126	0.436	-0.295	0.069	-0.074	0.650	0.385	0.069	0.650	m.m.m
6	0.105	0.516	0.105	0.516	0.221	0.173	0.402	0.173	0.516	m.m.m
7	-0.232	0.153	-0.011	0.948	0.084	0.604	0.568	0.153	0.948	m.l.e.
8	-0.221	0.173	-0.063	0.697	-0.168	0.299	0.390	0.173	0.697	m.l.e.
9	0.063	0.697	-0.295	0.069	0.179	0.270	0.345	0.069	0.697	m.l.e.
10	-0.011	0.948	-0.274	0.092	-0.126	0.436	0.492	0.092	0.948	m.m.m
11	-0.147	0.364	0.010	0.948	-0.063	0.697	0.670	0.364	0.948	m.l.e.
12	-0.116	0.475	-0.158	0.330	-0.095	0.559	0.455	0.330	0.559	m.l.e.
13	-0.042	0.795	-0.179	0.270	0.147	0.364	0.476	0.270	0.795	m.l.e.
14	-0.263	0.105	-0.179	0.270	-0.179	0.270	0.215	0.105	0.270	m.m.m
15	-0.168	0.299	0.232	0.153	-0.326	0.044	0.166	0.044	0.299	m.m.m
16	0.189	0.243	0.221	0.173	-0.021	0.897	0.438	0.173	0.897	m.l.e.
17	-0.126	0.436	-0.400	0.014	0.116	0.475	0.308	0.014	0.475	m.l.e.
18	0.105	0.516	0.053	0.746	0.105	0.746	0.669	0.516	0.746	m.l.e.
19	-0.126	0.436	-0.063	0.697	0.011	0.948	0.694	0.436	0.948	m.m.m
20	-0.116	0.475	0.221	0.173	-0.200	0.218	0.289	0.173	0.475	m.l.e.
21	-0.168	0.218	0.274	0.092	-0.095	0.559	0.289	0.092	0.559	m.m.m
22	-0.211	0.194	-0.095	0.559	0.042	0.795	0.516	0.194	0.795	m.l.e.
23	-0.189	0.243	0.242	0.136	-0.042	0.795	0.391	0.136	0.795	m.l.e.
24	-0.074	0.650	-0.116	0.475	0.032	0.846	0.657	0.475	0.846	m.l.e.
25	0.147	0.364	-0.253	0.119	-0.053	0.746	0.410	0.119	0.746	m.l.e.
26	-0.042	0.795	0.021	0.897	-0.053	0.746	0.813	0.746	0.897	m.m.m
27	-0.074	0.650	0.147	0.364	0.105	0.516	0.510	0.364	0.650	m.m.m
28	-0.042	0.795	-0.263	0.105	-0.084	0.604	0.501	0.105	0.795	m.l.e.
29	-0.074	0.650	-0.105	0.516	-0.105	0.516	0.561	0.516	0.650	m.l.e.
30	-0.095	0.559	0.032	0.846	0.032	0.846	0.750	0.559	0.846	m.l.e.
	mean	**mean**	**mean**	**mean**	**mean**	**mean**	**mean**	**mean**	**mean**	
	-0.08804	0.45767	-0.04211	0.38100	-0.01714	0.55823	0.46563	0.24566	0.69414	
	stdev	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	
	0.11499	0.22407	0.19334	0.29522	0.12607	0.23159	0.15545	0.18088	0.18533	

Table 10. Kendall -Tau Test - 100 Engines

KENDALL-TAU TEST OF INDEPENDENCE FOR THE SET OF 100 ENGINES

Replication Number	tau12	pvalue	tau13	pvalue	tau23	pvalue	mean	min	max	best method
1	0.029	0.668	-0.136	0.045	-0.072	0.292	0.335	0.045	0.668	m.l.e.
2	-0.093	0.173	-0.181	0.008	0.003	0.967	0.382	0.008	0.967	m.l.e.
3	-0.181	0.008	0.003	0.967	-0.025	0.708	0.561	0.008	0.967	m.l.e.
4	-0.144	0.034	-0.792	0.428	-0.064	0.347	0.270	0.034	0.428	m.m.m
5	-0.089	0.190	0.088	0.192	-0.036	0.592	0.325	0.190	0.592	m.m.m
6	-0.138	0.042	-0.166	0.015	-0.098	0.150	0.069	0.015	0.150	m.m.m
7	-0.028	0.681	-0.059	0.381	-0.064	0.344	0.469	0.344	0.681	m.l.e.
8	-0.036	0.592	-0.053	0.435	-0.145	0.033	0.353	0.033	0.592	m.m.m
9	-0.025	0.708	-0.009	0.896	-0.109	0.108	0.570	0.108	0.896	m.l.e.
10	-0.026	0.703	-0.110	0.104	-0.019	0.775	0.527	0.104	0.775	m.m.m
11	-0.002	0.976	0.042	0.536	0.026	0.703	0.738	0.536	0.976	m.l.e.
12	-0.132	0.052	0.064	0.347	-0.017	0.798	0.399	0.052	0.798	m.l.e.
13	-0.181	0.008	0.003	0.967	-0.025	0.708	0.561	0.008	0.967	m.l.e.
14	-0.081	0.231	-0.069	0.308	-0.018	0.789	0.443	0.231	0.789	m.l.e.
15	-0.051	0.449	0.095	0.162	-0.093	0.173	0.261	0.162	0.449	m.m.m
16	-0.250	0.000	-0.060	0.378	-0.039	0.568	0.315	0.000	0.568	m.l.e.
17	-0.070	0.303	0.011	0.868	-0.158	0.020	0.397	0.020	0.868	m.l.e.
18	-0.059	0.385	0.016	0.816	-0.004	0.953	0.718	0.385	0.953	m.m.m
19	-0.074	0.273	-0.053	0.435	-0.134	0.048	0.252	0.048	0.435	m.l.e.
20	-0.055	0.415	-0.022	0.748	-0.023	0.739	0.634	0.415	0.748	m.l.e.
21	-0.058	0.394	-0.120	0.077	-0.068	0.314	0.262	0.077	0.394	m.m.m
22	-0.104	0.126	0.122	0.072	-0.198	0.004	0.067	0.004	0.126	m.l.e.
23	-0.046	0.501	-0.161	0.018	-0.135	0.046	0.188	0.018	0.501	m.l.e.
24	-0.093	0.173	-0.118	0.082	-0.058	0.391	0.215	0.082	0.391	m.l.e.
25	0.022	0.748	0.071	0.297	-0.039	0.568	0.538	0.297	0.748	m.l.e.
26	-0.031	0.651	0.041	0.544	-0.210	0.002	0.399	0.002	0.651	m.l.e.
27	0.006	0.929	-0.051	0.457	-0.002	0.976	0.787	0.457	0.976	m.l.e.
28	-0.029	0.668	-0.005	0.938	0.017	0.803	0.803	0.668	0.938	m.m.m
29	-0.160	0.019	-0.022	0.743	-0.020	0.770	0.511	0.019	0.770	m.l.e.
30	-0.189	0.005	0.002	0.981	-0.154	0.024	0.337	0.005	0.981	m.m.m
	mean	**mean**	**mean**	**mean**	**mean**	**mean**	**mean**	**mean**	**mean**	
	-0.07884	0.37010	-0.05429	0.44144	-0.06601	0.45688	0.42281	0.14569	0.69136	
	stdev	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	**stdev**	
	0.06784	0.30264	0.16079	0.33413	0.06379	0.33785	0.19490	0.18499	0.24739	

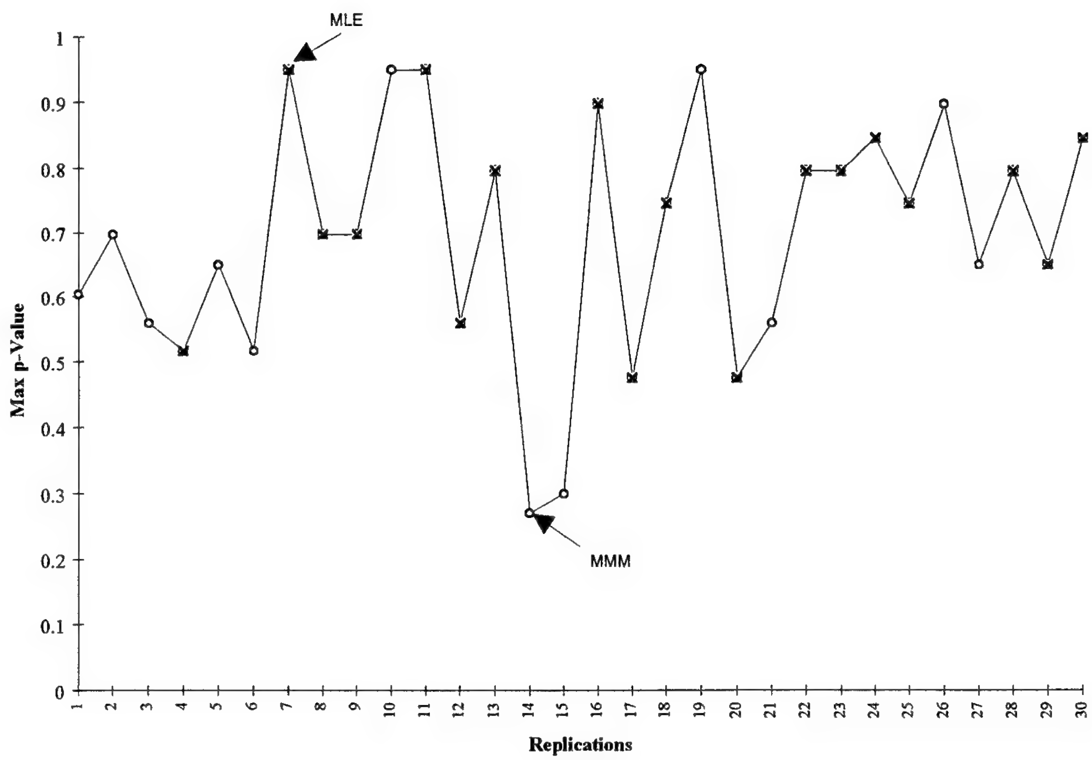

Figure 1. Maximum p-Value for 20 Engines

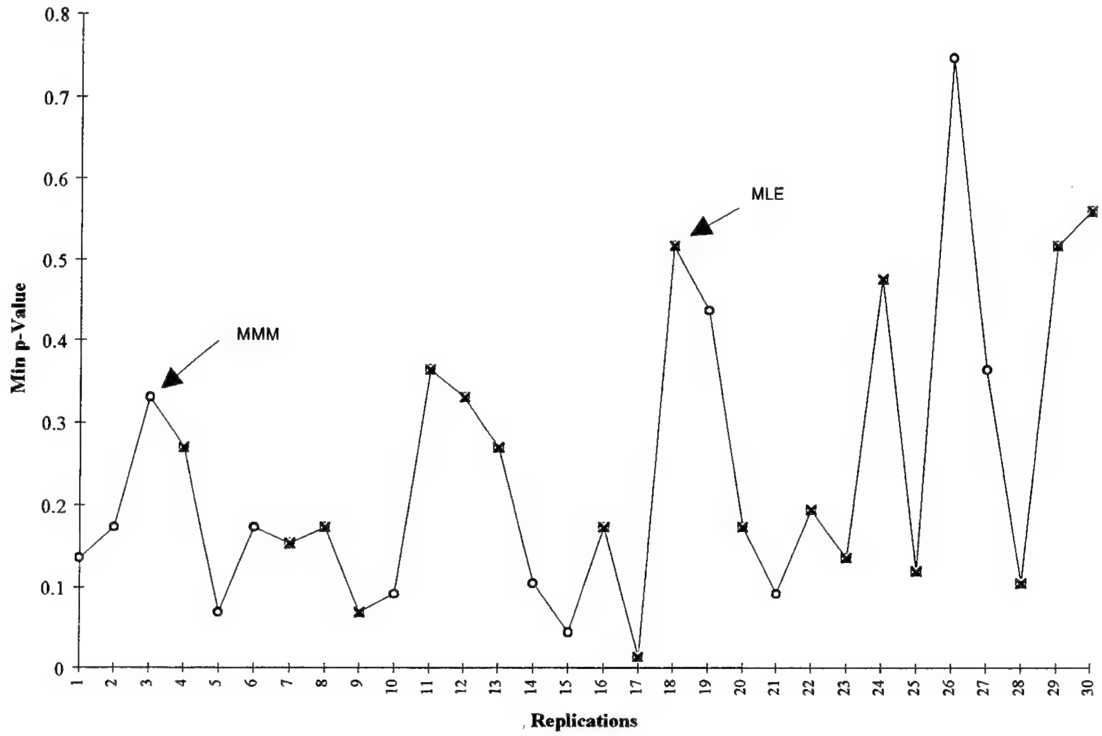

Figure 2. Minimum p-Value for 20 Engines

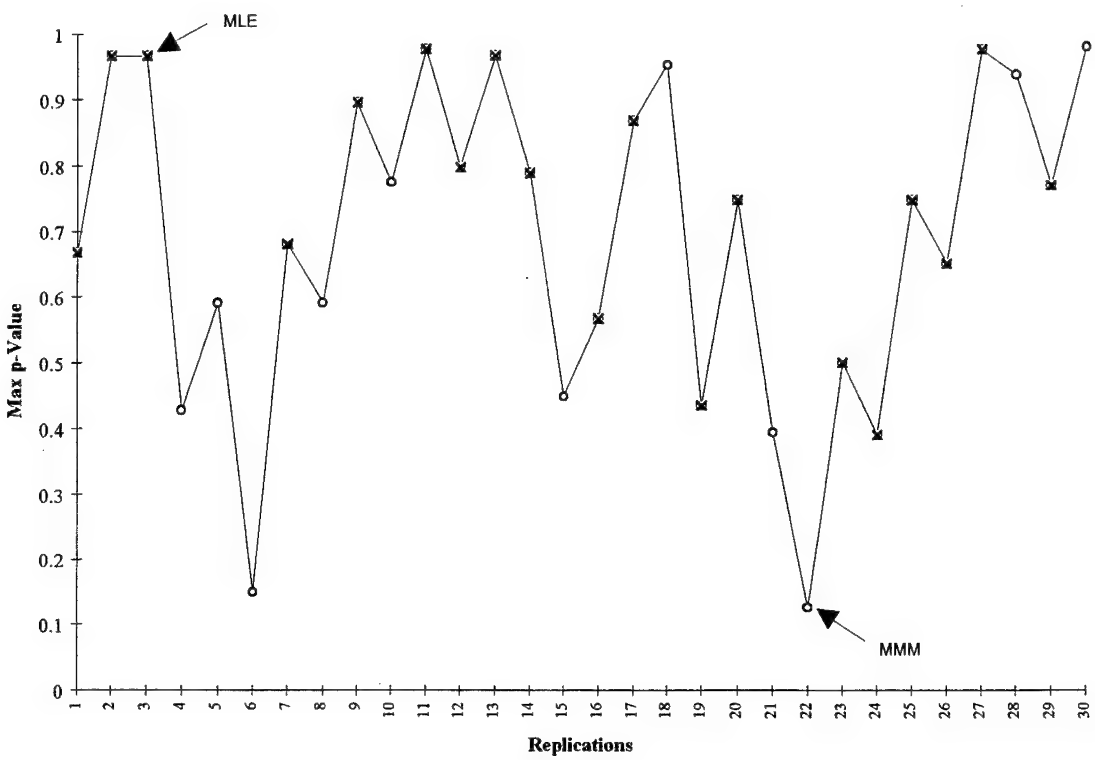

Figure 3. Maximum p-Value for 100 Engines

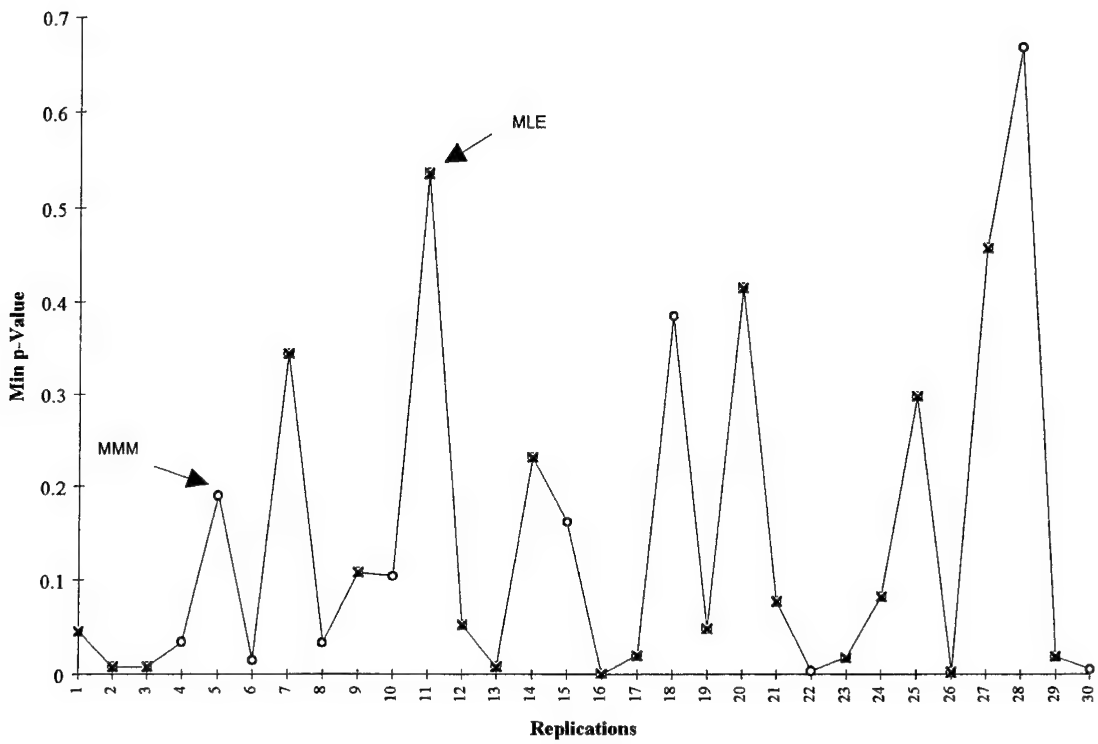

Figure 4. Minimum p-Value for 100 Engines

V. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

The analytical approach taken to develop this thesis and topics covered has been the following:

1. Develop a program in S-Plus to generate the failure times for an engine with 3 subsystems, where the distribution of the first failure times of these subsystems is Weibull and the engine is not restored to new condition after each engine failure.

2. Estimate the parameters for the Weibull distribution that fits the generated engine failure times with the Maximum Likelihood Estimator and a Modified Method of the Moments using commercial spread-sheet software.

3. Measure the accuracy of these two methods with the Kolmogorov-Smirnov Test.

4. Apply the same analysis to a given set of real engine failure times.

5. Compare the accuracy of the two methods, and discuss reasons for their differences.

B. CONCLUSIONS

The two methods in general give good results when they are applied to fit a Weibull distribution to the failure times generated in the simulation. The resulting fit for these failure times was better with the Maximum Likelihood Estimator more often than with the Modified Moment Method. For this reason, and the fact that the MLE method has superior theoretical properties in the presence of independence, the author recommends the MLE method to estimate the parameters for scenarios similar to those simulated in this Thesis.

With these conclusions it is important to point out the following considerations:

The MMM has the advantage that fewer calculations are necessary to get the results. In the case of the Macros developed in this study, only the initial values and two columns of calculations are necessary. Four columns of calculations are necessary in the MLE case.

There was a wide range of values for the parameters of the fitted distributions across all the replications. This is displayed in Table 11.

Table 11. Parameters Range

20 Engines	
MLE	**MMM**
0.9611 <= shape <= 1.5238	0.9857 <= shape <= 1.4616
223.28 <= scale <= 330.53	236.97 <= scale <= 329.43
100 Engines	
MLE	**MMM**
1.1002 <= shape <= 1,3422	1.1737 <= shape <= 1.4485
255.64 <= scale <= 301.25	254.07 <= scale <= 305.76

The dependence obtained in the simulation is not as significant as was expected.

C. RECOMMENDATIONS

These two methods should be studied when greater dependence exists among the data and when the dependency is introduced in a different manner than used in this thesis.

APPENDIX A. PROGRAM TO GENERATE FAILURE TIMES

```
function(n)
{
# generate
# This function generates times randomly in a manner that resembles the way in which
# engines failures occur in the real world.
        totalreplic <- NULL
        for(count in 1:n) {
# Step 1
# Three Weibull random numbers are generated, each one represents the first failure time
# that each one of the components could have. These values constitute the vector faults1.
# Three Weibull random numbers are generated, each one represents the second failure time
# that each one of the components could have. These values constitute the vector faults2.
# Three Weibull random numbers are generated, each one represents the third failure time
# that each one of the components could have. These values constitute the vector faults3.
        x11 <- rweibull(1, 1.2, 900)
        x12 <- rweibull(1, 1.3, 800)
        x13 <- rweibull(1, 1.4, 600)
        faults1 <- c(x11, x12, x13)
        x21 <- rweibull(1, 1.2, 900)
        x22 <- rweibull(1, 1.3, 800)
        x23 <- rweibull(1, 1.4, 600)
        faults2 <- c(x21, x22, x23)
        x31 <- rweibull(1, 1.2, 900)
        x32 <- rweibull(1, 1.3, 800)
        x33 <- rweibull(1, 1.4, 600)
        faults3 <- c(x31, x32, x33)
        failures <- matrix(c(faults1, faults2, faults3), ncol = 3)
# Step 2
# The minimum value of the vector faults1 is selected, this value is the time of the first
# failure of the engine. The program creates a new vector newfail2 that is the difference
# between the vector faults1 and the minimum value selected, it is clear that this new vector
# has a component that is 0, this component is replaced in the vector newfail2 for the
# element of the vector faults2 that match with the component which had the first failure.
# Choose the first failure time, that is the minimum
# time from the vector faults1
        f1 <- min(faults1)
# the next step is to choose the second failure time, to make this first
# the above value min(faults1) must be substituted for a new time
# generated in the vector faults2 and precisely in the same component.
# After this the new minimum is selected
        newfail2 <- (faults1 - f1)
        ii <- match(0, newfail2)
        newfail2[ii] <- faults2[ii]

# Step 3
# At this point the engine has two components that have accumulated working hours and
# one that has been just repaired. The vector newfail2 has two elements that have the
```

remainder of the theoretical first failure for these components and one element that is the
theoretical second failure for the other component. So, again, the minimum element of
this vector is selected and it is the second failure time of the engine.

```
        f2 <- min(newfail2)

# Step 4
# The program creates a new vector called newfail3 that is the difference between the vector
# newfail2 and this minimum value selected, it is clear that this vector has a component that
# is 0.

# Choose the third failure time, here there are two possible
#  cases, that the second failure is in the same component that
#   the first one or not, these two condition are programmed in this order
#
        newfail3 <- (newfail2 - f2)
        iii <- match(0, newfail3)
        if(ii == iii) {
                        newfail3[iii] <- faults3[iii]
                        f3 <- min(newfail3)
}
        else if(ii != iii) {
        newfail3[iii] <- faults2[iii]
                        f3 <- min(newfail3)
}
        enginefails <- c(f1, f2, f3)
      #cumenginefails <- cumsum(enginefails)
      #engfails <- matrix(c(failures, enginefails, cumenginefails), byrow =
      #   T, ncol = 3)
        totalreplic <- rbind(totalreplic, (enginefails))
}
        totalreplic
}
```

Row Names	V1	V2	V3
1	88.980852932	7.374101636	166.612433434
2	57.770945753	452.164077047	671.533220087
3	348.363054836	13.725217444	33.719418711
4	382.257870284	168.795667771	119.352260671
5	57.145359689	117.795729216	228.544288266
6	251.488400993	417.099715694	206.169150245
7	703.336492627	16.866142294	168.320579106
8	85.174113550	960.378668893	561.250692879
9	126.379607253	166.962891751	410.881188239
10	9.871149363	30.987527462	1171.995737937
11	184.016586935	55.183363300	170.715797827
12	19.085994032	206.177991742	124.656084035
13	171.708369596	7.411273439	458.366290751
14	957.877021060	334.941605556	252.553811069
15	88.483061291	129.532846338	55.052226186
16	97.662694658	64.764644477	521.223680534
17	213.546020387	8.955316669	314.364020817
18	420.340591055	182.036946622	345.506361499
19	46.823610226	411.857588415	109.450083602
20	25.300667331	361.149199434	744.757337329

	Column 1	*Column 2*	*Column 3*
Column 1	1		
Column 2	-0.099201099	1	
Column 3	-0.325682845	0.177625785	1

Row Names	V1	V2	V3
1	450.634670353	134.620284266	763.602013650
2	395.264314931	300.696009190	201.433889226
3	467.906036969	301.510351673	22.166715862
4	200.004796884	261.419709923	234.019837022
5	62.187250449	351.566316602	293.954533661
6	188.420555520	530.441439163	446.499347177
7	517.045829232	212.650323103	4.372290612
8	104.406621033	91.440692508	286.231002289
9	185.580658025	189.658931128	461.452317203
10	278.805459607	142.687073420	63.785901768
11	242.374076871	140.003133652	57.571537504
12	191.638384699	35.982496826	135.685578893
13	115.651429115	807.515319374	56.217725189
14	318.738490247	220.511514630	96.958868093
15	389.324028552	121.699393963	391.967785281
16	108.549643630	833.642731237	587.010089214
17	647.283140422	270.006356536	600.593877204
18	135.277879814	235.260576261	294.610909130
19	167.565552866	19.382052477	80.687152199
20	620.417102151	43.940840941	348.339229735
21	549.652520297	147.263663645	402.329146256
22	160.858623029	198.697751449	131.936073250
23	269.748636292	216.407382425	272.564254146
24	119.882883842	42.753769287	568.105938757
25	290.367131217	46.018913125	470.135356798
26	111.526180647	59.040162467	442.572648090
27	315.869668084	112.068427109	799.073849390
28	258.473619251	740.012832046	730.391096774
29	936.074568617	525.344669063	53.458836253
30	38.275293391	264.758222487	109.033459770
31	200.399299969	130.448227050	564.453719467
32	16.387049764	689.920420726	124.595174314
33	282.346963237	302.451537677	17.105846707
34	298.005804703	792.704752916	277.592370478
35	488.070794225	600.835486551	77.330402620
36	651.631129729	186.384979015	302.942800237
37	488.356688854	282.631614706	12.094239822
38	251.322489919	639.047414057	315.553538531
39	102.245119335	121.065856465	248.600567386
40	120.292743494	281.418670027	6.512039507
41	394.628880256	329.862437764	42.172683323
42	275.823361976	400.377074953	205.279868273
43	89.822151642	568.897212277	126.945901458
44	413.319725633	280.829982419	74.621453179
45	172.347938766	138.081515648	169.690890193
46	295.648493372	49.777202258	271.146965769
47	427.883569557	120.358845277	86.140070778
48	106.453003164	336.696295308	392.945334315
49	439.747178352	145.848786206	357.670884257
50	95.424425759	158.394437081	233.214847248

Row Names	V1	V2	V3
51	78.318115166	233.292392917	150.556841342
52	400.322169035	703.176710554	131.985760999
53	284.082094482	700.398323845	54.502213913
54	673.813459742	628.345709323	27.810511050
55	416.157442911	309.802781935	61.701311275
56	502.727434228	208.984463635	7.000471997
57	395.249167889	181.615779362	246.169600960
58	532.055233005	282.774380172	51.530633142
59	256.526079351	265.073575146	366.480188363
60	791.013013348	483.679778572	383.122947523
61	116.674678096	203.429431816	501.860086251
62	216.019262031	50.783576381	346.268316228
63	652.055312871	297.710549841	19.143684755
64	57.143580125	444.863422850	53.260846841
65	305.430041853	64.522458661	344.328881540
66	134.875969833	153.848996778	74.573546508
67	207.795329813	197.474623424	36.572283433
68	55.079317773	148.585617029	153.485311966
69	332.356937878	183.167123218	2.771444009
70	33.292174799	281.412899135	521.055842681
71	329.237256924	624.659923834	410.145804201
72	134.999172015	924.566774196	959.801158180
73	310.283780658	196.949468983	61.420855422
74	145.927518733	98.091275956	105.358948583
75	96.240971550	448.577731504	68.349212862
76	504.457185330	3.995392340	54.819812986
77	779.185987694	63.103630524	510.654006981
78	64.295309451	227.558484641	681.717084473
79	132.919226467	80.090112060	305.523448635
80	225.006666940	354.405022103	115.791651829
81	322.833702315	15.741543588	273.716914308
82	552.256221658	238.880493230	103.236641646
83	234.905588128	267.774447132	384.884639091
84	241.821803683	27.660947307	35.559153707
85	360.121368725	54.400309193	74.757012406
86	109.375105806	169.046509653	306.549361255
87	211.877188712	54.427577933	47.413167726
88	328.701690509	233.519929404	411.204384691
89	271.788277670	132.777355320	359.728088427
90	54.715071977	52.466467632	253.216046834
91	157.314280817	49.880943701	746.100150202
92	93.608619218	110.699690363	573.390687287
93	218.283430910	434.282741182	109.443120744
94	1106.554817220	178.424617471	107.752314459
95	179.703059410	96.191152623	63.123472845
96	220.753490182	358.476353401	438.979104011
97	97.987549536	57.556717482	152.972644520
98	126.992796611	173.350825362	1004.747325381
99	656.574251587	250.152057881	312.989740534
100	189.330584986	142.057479837	63.629475350

	Column 1	*Column 2*	*Column 3*
Column 1	1		
Column 2	0.012345873	1	
Column 3	-0.151985402	0.027275574	1

APPENDIX B. MACRO 20 ENGINES

```
' MACRO FOR 20 ENGINES
' thesis Macro
' Macro recorded  by FERNANDO GUIMIL
'
'
Sub thesis()
   Sheets.Add
   Sheets("Sheet2").Select
   Range("A1:C2").Select
   ActiveCell.FormulaR1C1 = "MAXIMUM LIKELIHOOD ESTIMATOR"
   Range("A1:C2").Select
   Range("A2").Activate
   Selection.Font.Bold = True
   With Selection
      .HorizontalAlignment = xlCenter
      .VerticalAlignment = xlBottom
      .WrapText = False
      .Orientation = xlHorizontal
   End With
   With Selection
      .HorizontalAlignment = xlCenterAcrossSelection
      .VerticalAlignment = xlBottom
      .WrapText = True
      .Orientation = xlHorizontal
   End With
   Range("D1").Select
   ActiveWindow.SmallScroll ToRight:=10
   With Toolbars(8)
      .Left = 241
      .Top = 228
   End With
   ActiveWindow.LargeScroll ToRight:=-1
   Range("P1:R1").Select
   ActiveCell.FormulaR1C1 = "MODIFIED MOMENT METHOD"
   Range("P1:R1").Select
   Range("Q1").Activate
   Selection.Font.Bold = True
   With Selection
      .HorizontalAlignment = xlCenter
      .VerticalAlignment = xlBottom
      .WrapText = False
      .Orientation = xlHorizontal
   End With
   With Selection
      .HorizontalAlignment = xlCenterAcrossSelection
      .VerticalAlignment = xlBottom
```

```
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("N1").Select
ActiveWindow.LargeScroll ToRight:=-1
With Toolbars(8)
    .Left = 436
    .Top = 226
End With
Range("A2").Select
ActiveCell.FormulaR1C1 = "shape parameter"
With ActiveCell.Characters(Start:=1, Length:=15).Font
    .Name = "Arial"
    .FontStyle = "Bold"
    .Size = 10
    .Strikethrough = False
    .Superscript = False
    .Subscript = False
    .OutlineFont = False
    .Shadow = False
    .Underline = xlNone
    .ColorIndex = xlAutomatic
End With
Range("B3").Select
ActiveCell.FormulaR1C1 = "initial beta"
Range("B3").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("B5").Select
ActiveCell.FormulaR1C1 = "final beta"
Range("B5").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("A7:C7").Select
ActiveCell.FormulaR1C1 = "scale parameter"
Range("A7:C7").Select
Range("B7").Activate
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
```

```
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("A7").Select
ActiveCell.FormulaR1C1 = ""
Range("B7").Select
ActiveCell.FormulaR1C1 = "scale parameter"
Range("B8").Select
ActiveCell.FormulaR1C1 = "landa"
Range("B8").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
ActiveWindow.SmallScroll ToRight:=0
Range("A2:C2").Select
Selection.Copy
ActiveWindow.SmallScroll ToRight:=11
Range("Q2").Select
ActiveSheet.Paste
Application.CutCopyMode = False
Selection.Clear
ActiveWindow.LargeScroll ToRight:=-2
Range("A2:C2").Select
Selection.Copy
ActiveWindow.SmallScroll ToRight:=9
Range("P2").Select
ActiveSheet.Paste
Range("O4").Select
ActiveWindow.LargeScroll ToRight:=-1
Range("B3").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=11
Range("Q3").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll ToRight:=-2
Range("B5").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=12
Range("Q5").Select
Application.CutCopyMode = False
Selection.Copy
ActiveSheet.Paste
Range("R7").Select
ActiveWindow.LargeScroll ToRight:=-2
Range("B5").Select
```

```
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=12
Range("Q5").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll ToRight:=-2
Range("A7:C8").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=11
Range("P7").Select
ActiveSheet.Paste
Range("N7").Select
ActiveWindow.LargeScroll ToRight:=-2
Range("D11").Select
Application.CutCopyMode = False
Selection.Copy
Range("D7").Select
ActiveSheet.Paste
Range("B8").Select
ActiveWindow.SmallScroll ToRight:=13
ActiveWindow.LargeScroll ToRight:=-2
ActiveWindow.SmallScroll ToRight:=14
ActiveWindow.LargeScroll ToRight:=-2
Range("E1").Select
Application.CutCopyMode = False
ActiveCell.FormulaR1C1 = "Number of observations"
Range("E1").Select
Selection.Font.Bold = True
With Selection
  .HorizontalAlignment = xlCenter
  .VerticalAlignment = xlBottom
  .WrapText = False
  .Orientation = xlHorizontal
End With
With Selection
  .HorizontalAlignment = xlCenterAcrossSelection
  .VerticalAlignment = xlBottom
  .WrapText = True
  .Orientation = xlHorizontal
End With
Range("E2").Select
ActiveCell.FormulaR1C1 = "n"
Range("E2").Select
With Toolbars(8)
  .Left = 536
  .Top = 23
End With
Selection.Font.Bold = True
With Selection
  .HorizontalAlignment = xlCenter
```

```
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Sheets("Sheet1").Select
Range("B2:B21").Select
Selection.Copy
Sheets("Sheet2").Select
Range("D10").Select
ActiveSheet.Paste
Sheets("Sheet1").Select
Range("C2:C21").Select
Application.CutCopyMode = False
Selection.Copy
Sheets("Sheet2").Select
Range("E10").Select
ActiveSheet.Paste
Sheets("Sheet1").Select
Range("D2:D21").Select
Application.CutCopyMode = False
Selection.Copy
Sheets("Sheet2").Select
Range("F10").Select
Application.CutCopyMode = False
Selection.Copy
Sheets("Sheet1").Select
Application.CutCopyMode = False
Selection.Copy
Sheets("Sheet2").Select
ActiveSheet.Paste
Range("D10:D29").Select
Application.CutCopyMode = False
Selection.Copy
Range("G10").Select
ActiveSheet.Paste
Range("E10:E29").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll Down:=16
Range("G30").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll Down:=-1
Range("F10:F29").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll Down:=35
Range("G50").Select
ActiveSheet.Paste
Range("D42").Select
ActiveWindow.LargeScroll Down:=-2
Range("G10:G69").Select
```

```
Application.CutCopyMode = False
Selection.Copy
Range("H10").Select
ActiveSheet.Paste
Application.CutCopyMode = False
Selection.Sort Key1:=Range("H10"), Order1:=xlAscending, Header:= _
    xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:= _
    xlTopToBottom
Range("B10").Select
ActiveCell.FormulaR1C1 = "theta"
Range("B10").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("E3").Select
ActiveCell.FormulaR1C1 = "60"
Range("E3").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
ActiveWindow.ScrollRow = 1
Range("G1").Select
ActiveCell.FormulaR1C1 = "Failures times Ti"
Range("G1").Select
With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
End With
Selection.Font.Bold = True
Range("G9").Select
ActiveCell.FormulaR1C1 = "Ti"
Range("G9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Selection.Copy
Range("H9").Select
```

```
ActiveSheet.Paste
Range("G9").Select
Application.CutCopyMode = False
Selection.Clear
ActiveWindow.SmallScroll ToRight:=3
Range("I9").Select
ActiveCell.FormulaR1C1 = "Ln Ti"
Range("I9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("I10").Select
ActiveCell.FormulaR1C1 = "=LN(RC[-1])"
Range("I10").Select
Selection.AutoFill Destination:=Range("I10:I69"), Type:= _
    xlFillDefault
Range("I10:I69").Select
Range("H70").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("H70").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Selection.Copy
Range("I70").Select
ActiveSheet.Paste
Range("H71").Select
Application.CutCopyMode = False
ActiveCell.FormulaR1C1 = "=SUM(R[-61]C:R[-2]C)"
Range("I71").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-61]C:R[-2]C)"
Range("I72").Select
ActiveWindow.LargeScroll Down:=-3
Range("J9").Select
ActiveCell.FormulaR1C1 = "Ti^B4"
Range("J9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
```

```
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=2
Range("J10").Select
ActiveCell.FormulaR1C1 = "=RC[-2]^R[-6]C[-8]"
Range("J10").Select
Selection.AutoFill Destination:=Range("J10:J69"), Type:= _
    xlFillDefault
Range("J10:J69").Select
Selection.Replace What:="^B*", Replacement:="^B4", LookAt:=xlPart, _
    SearchOrder:=xlByRows, MatchCase:=False
Range("K9").Select
ActiveWindow.SmallScroll ToRight:=2
ActiveCell.FormulaR1C1 = "Ti^B6"
Range("K9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("K10").Select
ActiveCell.FormulaR1C1 = "=RC[-3]^R[-4]C[-9]"
Range("K10").Select
Selection.AutoFill Destination:=Range("K10:K69"), Type:= _
    xlFillDefault
Range("K10:K69").Select
Selection.Replace What:="^B*", Replacement:="^B6", LookAt:=xlPart, _
    SearchOrder:=xlByRows, MatchCase:=False
Range("L9").Select
ActiveCell.FormulaR1C1 = "Ln TI*Ti^B6"
Range("L9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("L9").Select
ActiveCell.FormulaR1C1 = "Ln TI * Ti^B6"
With ActiveCell.Characters(Start:=1, Length:=13).Font
    .Name = "Arial"
    .FontStyle = "Bold"
    .Size = 10
    .Strikethrough = False
    .Superscript = False
    .Subscript = False
    .OutlineFont = False
    .Shadow = False
    .Underline = xlNone
```

```
    .ColorIndex = xlAutomatic
  End With
  Range("L10").Select
  ActiveCell.FormulaR1C1 = "=RC[-3]*RC[-1]"
  Range("L10").Select
  Selection.AutoFill Destination:=Range("L10:L69"), Type:= _
    xlFillDefault
  Range("L10:L69").Select
  Range("M64").Select
  ActiveWindow.LargeScroll Down:=-3
  ActiveWindow.SmallScroll Down:=12
  ActiveWindow.ScrollRow = 28
  ActiveWindow.ScrollRow = 1
  ActiveWindow.ScrollRow = 61
  Range("K70").Select
  ActiveCell.FormulaR1C1 = "SUM"
  Range("K70").Select
  Selection.Font.Bold = True
  With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
  End With
  Selection.Copy
  Range("L70").Select
  ActiveSheet.Paste
  Range("K71").Select
  Application.CutCopyMode = False
  ActiveCell.FormulaR1C1 = "=SUM(R[-61]C:R[-2]C)"
  Range("L71").Select
  ActiveCell.FormulaR1C1 = "=SUM(R[-61]C:R[-2]C)"
  Range("L72").Select
  With Toolbars(8)
    .Left = 57
    .Top = 106
  End With
  ActiveWindow.LargeScroll Down:=-4
  Range("M9").Select
  ActiveCell.FormulaR1C1 = "Ti^Q6"
  Range("M9").Select
  Selection.Font.Bold = True
  With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
  End With
  Range("M10").Select
  ActiveCell.FormulaR1C1 = "=RC[-5]^R[-4]C[4]"
  Range("M10").Select
```

```
Selection.AutoFill Destination:=Range("M10:M69"), Type:= _
    xlFillDefault
Range("M10:M69").Select
Selection.Replace What:="^Q*", Replacement:="^Q6", LookAt:=xlPart, _
    SearchOrder:=xlByRows, MatchCase:=False
ActiveWindow.SmallScroll Down:=8
ActiveWindow.ScrollRow = 33
ActiveWindow.ScrollRow = 43
ActiveWindow.ScrollRow = 48
ActiveWindow.ScrollRow = 51
ActiveWindow.ScrollRow = 57
Range("M70").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("M70").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("M71").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-61]C:R[-2]C)"
Range("M72").Select
ActiveCell.FormulaR1C1 = "mean"
Range("M73").Select
ActiveCell.FormulaR1C1 = ""
Range("M72").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("M73").Select
ActiveCell.FormulaR1C1 = "=R[-2]C/60"
Range("M74").Select
ActiveWindow.ScrollRow = 1
ActiveWindow.SmallScroll ToRight:=2
Range("N9").Select
ActiveCell.FormulaR1C1 = "(Ti^Q6-mean)^2"
Range("N9").Select
With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
End With
Selection.Font.Bold = True
ActiveWindow.SmallScroll Down:=59
```

```
ActiveWindow.LargeScroll Down:=-4
Range("N10").Select
ActiveCell.FormulaR1C1 = "=(RC[-1]-R[63]C[-1])^2"
Range("N10").Select
Selection.AutoFill Destination:=Range("N10:N69"), Type:= _
   xlFillDefault
Range("N10:N69").Select
Selection.Replace What:="-M*)^2", Replacement:="-M73)^2", LookAt _
   :=xlPart, SearchOrder:=xlByRows, MatchCase:=False
Range("O9").Select
ActiveCell.FormulaR1C1 = "(Ln Ti)^2"
Range("O9").Select
Selection.Font.Bold = True
With Selection
   .HorizontalAlignment = xlCenter
   .VerticalAlignment = xlBottom
   .WrapText = False
   .Orientation = xlHorizontal
End With
Range("O10").Select
ActiveCell.FormulaR1C1 = "=RC[-6]^2"
Range("O10").Select
Selection.AutoFill Destination:=Range("O10:O69"), Type:= _
   xlFillDefault
Range("O10:O69").Select
Range("M70").Select
Selection.Copy
Range("N70").Select
ActiveSheet.Paste
Range("O70").Select
ActiveSheet.Paste
Range("N71").Select
Application.CutCopyMode = False
ActiveCell.FormulaR1C1 = "=SUM(R[-61]C:R[-2]C)"
Range("O71").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-61]C:R[-2]C)"
Range("O72").Select
ActiveWindow.SmallScroll ToRight:=2
ActiveWindow.LargeScroll Down:=-3
ActiveWindow.ScrollRow = 1
ActiveWindow.LargeScroll ToRight:=-1
With Toolbars(8)
   .Left = 543
   .Top = 37
End With
Range("B4").Select
ActiveWindow.SmallScroll ToRight:=6
ActiveWindow.SmallScroll Down:=60
ActiveWindow.LargeScroll Down:=-4
ActiveWindow.LargeScroll ToRight:=-1
Range("B4").Select
```

```
    ActiveCell.FormulaR1C1 = _
        "=((6/(PI()^2))*(R[67]C[13]-((R[67]C[7]^2)/60))*(1/59))^(-0.5)"
    Range("B4").Select
    Selection.Copy
    ActiveWindow.SmallScroll ToRight:=10
    Range("Q4").Select
    ActiveSheet.Paste
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = _
        "=((6/(PI()^2))*(R[67]C[-2]-((R[67]C[-8]^2)/60))*(1/59))^(-0.5)"
    Range("Q5").Select
    ActiveWindow.LargeScroll ToRight:=-1
    ActiveWindow.SmallScroll ToRight:=-1
    Range("C6").Select
    ActiveCell.FormulaR1C1 = _
        "=(60/RC[-1])+R[65]C[6]-(60*R[65]C[9]/R[65]C[8])"
    Range("C7").Select
    ActiveWindow.LargeScroll ToRight:=-1
    Range("B9").Select
    ActiveCell.FormulaR1C1 = "=(60/R[62]C[9])^(1/R[-3]C)"
    Range("B10").Select
    ActiveWindow.SmallScroll ToRight:=-1
    Range("B11").Select
    ActiveCell.FormulaR1C1 = "=1/R[-2]C"
    Range("B12").Select
    ActiveWindow.SmallScroll ToRight:=11
    Range("P6").Select
    ActiveCell.FormulaR1C1 = "=(R[65]C[-3])/((R[65]C[-2])^(0.5))"
    Range("Q9").Select
    ActiveWindow.LargeScroll ToRight:=-2
    ActiveWindow.SmallScroll ToRight:=9
    Range("Q10").Select
    ActiveCell.FormulaR1C1 = "theta"
    Range("Q10").Select
    Selection.Font.Bold = True
    With Selection
        .HorizontalAlignment = xlCenter
        .VerticalAlignment = xlBottom
        .WrapText = False
        .Orientation = xlHorizontal
    Range("Q9").Select
    ActiveCell.FormulaR1C1 = "=1/(((1/60)*R[62]C[-4])^(1/R[-3]C))"
    Range("Q11").Select
    ActiveCell.FormulaR1C1 = "=1/R[-2]C"
    Range("Q12").Select
    End With


End Sub
```

APPENDIX C. MACRO 100 ENGINES

```
' MACRO TO 100 ENGINES
' thesis Macro
' Macro recorded  by FERNANDO GUIMIL
'
'
Sub thesis()
  Sheets.Add
  Sheets("Sheet2").Select
  Range("A1:C2").Select
  ActiveCell.FormulaR1C1 = "MAXIMUM LIKELIHOOD ESTIMATOR"
  Range("A1:C2").Select
  Range("A2").Activate
  Selection.Font.Bold = True
  With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
  End With
  With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
  End With
  Range("D1").Select
  ActiveWindow.SmallScroll ToRight:=10
  With Toolbars(8)
    .Left = 241
    .Top = 228
  End With
  ActiveWindow.LargeScroll ToRight:=-1
  Range("P1:R1").Select
  ActiveCell.FormulaR1C1 = "MODIFIED MOMENT METHOD"
  Range("P1:R1").Select
  Range("Q1").Activate
  Selection.Font.Bold = True
  With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
  End With
  With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
```

```
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("N1").Select
ActiveWindow.LargeScroll ToRight:=-1
With Toolbars(8)
    .Left = 436
    .Top = 226
End With
Range("A2").Select
ActiveCell.FormulaR1C1 = "shape parameter"
With ActiveCell.Characters(Start:=1, Length:=15).Font
    .Name = "Arial"
    .FontStyle = "Bold"
    .Size = 10
    .Strikethrough = False
    .Superscript = False
    .Subscript = False
    .OutlineFont = False
    .Shadow = False
    .Underline = xlNone
    .ColorIndex = xlAutomatic
End With
Range("B3").Select
ActiveCell.FormulaR1C1 = "initial beta"
Range("B3").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("B5").Select
ActiveCell.FormulaR1C1 = "final beta"
Range("B5").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("A7:C7").Select
ActiveCell.FormulaR1C1 = "scale parameter"
Range("A7:C7").Select
Range("B7").Activate
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
```

```
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("A7").Select
ActiveCell.FormulaR1C1 = ""
Range("B7").Select
ActiveCell.FormulaR1C1 = "scale parameter"
Range("B8").Select
ActiveCell.FormulaR1C1 = "landa"
Range("B8").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
ActiveWindow.SmallScroll ToRight:=0
Range("A2:C2").Select
Selection.Copy
ActiveWindow.SmallScroll ToRight:=11
Range("Q2").Select
ActiveSheet.Paste
Application.CutCopyMode = False
Selection.Clear
ActiveWindow.LargeScroll ToRight:=-2
Range("A2:C2").Select
Selection.Copy
ActiveWindow.SmallScroll ToRight:=9
Range("P2").Select
ActiveSheet.Paste
Range("O4").Select
ActiveWindow.LargeScroll ToRight:=-1
Range("B3").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=11
Range("Q3").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll ToRight:=-2
Range("B5").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=12
Range("Q5").Select
Application.CutCopyMode = False
Selection.Copy
ActiveSheet.Paste
Range("R7").Select
ActiveWindow.LargeScroll ToRight:=-2
Range("B5").Select
```

```
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=12
Range("Q5").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll ToRight:=-2
Range("A7:C8").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll ToRight:=11
Range("P7").Select
ActiveSheet.Paste
Range("N7").Select
ActiveWindow.LargeScroll ToRight:=-2
Range("D11").Select
Application.CutCopyMode = False
Selection.Copy
Range("D7").Select
ActiveSheet.Paste
Range("B8").Select
ActiveWindow.SmallScroll ToRight:=13
ActiveWindow.LargeScroll ToRight:=-2
ActiveWindow.SmallScroll ToRight:=14
ActiveWindow.LargeScroll ToRight:=-2
Range("E1").Select
Application.CutCopyMode = False
ActiveCell.FormulaR1C1 = "Number of observations"
Range("E1").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("E2").Select
ActiveCell.FormulaR1C1 = "n"
Range("E2").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
```

```
With Toolbars(8)
  .Left = 539
  .Top = 30
End With
Sheets("Sheet1").Select
Range("B2:B101").Select
Selection.Copy
Sheets("Sheet2").Select
ActiveWindow.SmallScroll Down:=1
Range("D10").Select
ActiveSheet.Paste
Sheets("Sheet1").Select
Range("C2:C101").Select
Application.CutCopyMode = False
Selection.Copy
Sheets("Sheet2").Select
Range("E10").Select
ActiveSheet.Paste
Sheets("Sheet1").Select
Range("D2:D101").Select
Application.CutCopyMode = False
Selection.Copy
Sheets("Sheet2").Select
Range("F10").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll ToRight:=0
Range("D10:D109").Select
Application.CutCopyMode = False
Selection.Copy
Range("G10").Select
ActiveSheet.Paste
Range("E10:E109").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll Down:=100
Range("G110").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll Down:=-6
Range("F10:F109").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.SmallScroll Down:=200
Range("G210").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll Down:=-11
Range("G10:G309").Select
Application.CutCopyMode = False
Selection.Copy
Range("H10").Select
ActiveSheet.Paste
Application.CutCopyMode = False
```

```
Selection.Sort Key1:=Range("H10"), Order1:=xlAscending, Header:= _
    xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:= _
    xlTopToBottom
ActiveWindow.SmallScroll Down:=-1
Range("H2:H6").Select
ActiveCell.FormulaR1C1 = "failures times Ti"
Range("H2:H6").Select
Range("H3").Activate
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("H9").Select
ActiveCell.FormulaR1C1 = "Ti"
Range("H9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("I9").Select
ActiveCell.FormulaR1C1 = "Ln Ti"
Range("I9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("I10").Select
ActiveCell.FormulaR1C1 = "=LN(RC[-1])"
Range("I10").Select
Selection.AutoFill Destination:=Range("I10:I309"), Type:= _
    xlFillDefault
Range("I10:I309").Select
With Toolbars(8)
    .Left = 79
    .Top = 270
End With
```

```
Range("G315").Select
ActiveWindow.SmallScroll ToRight:=2
Range("I305").Select
ActiveWindow.LargeScroll Down:=-17
Range("J9").Select
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=2
Range("J9").Select
ActiveCell.FormulaR1C1 = "Ti^B4"
Range("J9").Select
Selection.Font.Bold = True
With Selection
   .HorizontalAlignment = xlCenter
   .VerticalAlignment = xlBottom
   .WrapText = False
   .Orientation = xlHorizontal
End With
Selection.Copy
Range("K9").Select
ActiveSheet.Paste
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=4
Application.CutCopyMode = False
ActiveCell.FormulaR1C1 = "Ti^B6"
Range("J10").Select
ActiveCell.FormulaR1C1 = "=(RC[-2]^R[-6]C[-8])"
Range("J10").Select
Selection.AutoFill Destination:=Range("J10:J309"), Type:= _
   xlFillDefault
Range("J10:J309").Select
Range("J300").Select
ActiveWindow.LargeScroll Down:=-16
ActiveWindow.SmallScroll Down:=-9
Range("J10:J309").Select
Selection.Replace What:="^B*)", Replacement:="^B4)", LookAt:= _
   xlPart, SearchOrder:=xlByRows, MatchCase:=False
Range("K10").Select
ActiveCell.FormulaR1C1 = "=(RC[-3]^R[-4]C[-9])"
Range("K10").Select
Selection.AutoFill Destination:=Range("K10:K309"), Type:= _
   xlFillDefault
Range("K10:K309").Select
Selection.Replace What:="^B*)", Replacement:="^B6)", LookAt:= _
   xlPart, SearchOrder:=xlByRows, MatchCase:=False
Range("L8:L9").Select
ActiveCell.FormulaR1C1 = "Ln Ti * Ti^B6"
Range("L8:L9").Select
Range("L9").Activate
With Selection
   .HorizontalAlignment = xlCenterAcrossSelection
   .VerticalAlignment = xlBottom
```

```
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("L9").Select
Selection.Delete Shift:=xlUp
Range("L8").Select
Selection.Copy
Range("L9").Select
ActiveSheet.Paste
Range("L8").Select
Application.CutCopyMode = False
Selection.Clear
Range("L9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("L10").Select
ActiveCell.FormulaR1C1 = "=RC[-3]*RC[-1]"
Range("L10").Select
Selection.AutoFill Destination:=Range("L10:L309"), Type:= _
    xlFillDefault
Range("L10:L309").Select
ActiveWindow.LargeScroll Down:=-1
Range("M285").Select
ActiveWindow.LargeScroll Down:=-16
ActiveWindow.SmallScroll ToRight:=4
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=8
ActiveWindow.SmallScroll Down:=33
ActiveWindow.LargeScroll Down:=-2
ActiveWindow.ScrollRow = 174
ActiveWindow.ScrollRow = 218
ActiveWindow.ScrollRow = 252
ActiveWindow.ScrollRow = 309
Range("L310").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("L310").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("L311").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-301]C:R[-2]C)"
Range("L312").Select
```

```
Range("M9").Select
ActiveCell.FormulaR1C1 = "Ti^Q6"
Range("M9").Select
Selection.Font.Bold = True
With Selection
   .HorizontalAlignment = xlCenter
   .VerticalAlignment = xlBottom
   .WrapText = False
   .Orientation = xlHorizontal
End With
Range("M10").Select
ActiveCell.FormulaR1C1 = "=(RC[-5]^R[-4]C[4])"
Range("M10").Select
Selection.AutoFill Destination:=Range("M10:M309"), Type:= _
   xlFillDefault
Range("M10:M309").Select
Selection.Replace What:="^Q*)", Replacement:="^Q6)", LookAt:= _
   xlPart, SearchOrder:=xlByRows, MatchCase:=False
Range("N9").Select
ActiveWindow.SmallScroll Down:=107
ActiveWindow.ScrollRow = 162
ActiveWindow.ScrollRow = 214
ActiveWindow.ScrollRow = 243
ActiveWindow.ScrollRow = 261
ActiveWindow.ScrollRow = 282
ActiveWindow.ScrollRow = 297
Range("M310").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("M310").Select
Selection.Font.Bold = True
With Selection
   .HorizontalAlignment = xlCenter
   .VerticalAlignment = xlBottom
   .WrapText = False
   .Orientation = xlHorizontal
End With
Range("M311").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-301]C:R[-2]C)"
Range("M312").Select
ActiveCell.FormulaR1C1 = "mean"
Range("M312").Select
Selection.Font.Bold = True
With Selection
   .HorizontalAlignment = xlCenter
   .VerticalAlignment = xlBottom
   .WrapText = False
   .Orientation = xlHorizontal
End With
Range("M313").Select
ActiveWindow.LargeScroll Down:=-17
Range("H3").Select
```

```
ActiveWindow.LargeScroll ToRight:=-1
Range("E3").Select
ActiveCell.FormulaR1C1 = "300"
Range("E3").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
ActiveWindow.SmallScroll ToRight:=5
ActiveWindow.ScrollRow = 144
ActiveWindow.ScrollRow = 178
ActiveWindow.ScrollRow = 219
ActiveWindow.ScrollRow = 264
ActiveWindow.ScrollRow = 291
ActiveWindow.ScrollRow = 312
ActiveWindow.ScrollRow = 274
ActiveWindow.LargeScroll Down:=-15
ActiveWindow.SmallScroll Down:=37
ActiveWindow.ScrollRow = 283
ActiveWindow.ScrollRow = 312
ActiveWindow.ScrollRow = 303
Range("M313").Select
ActiveCell.FormulaR1C1 = "=R[-2]C/300"
Range("M314").Select
ActiveWindow.LargeScroll Down:=-17
Range("N9").Select
ActiveCell.FormulaR1C1 = "(Ti^Q6-mean)^2"
Range("N9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
With Selection
    .HorizontalAlignment = xlCenterAcrossSelection
    .VerticalAlignment = xlBottom
    .WrapText = True
    .Orientation = xlHorizontal
End With
Range("N10").Select
ActiveCell.FormulaR1C1 = "=(RC[-2]-R[303]C[-1])"
Range("N10").Select
Selection.AutoFill Destination:=Range("N10:N309"), Type:= _
    xlFillDefault
Range("N10:N309").Select
ActiveWindow.SmallScroll ToRight:=1
```

```
Range("N10").Select
ActiveCell.FormulaR1C1 = "=(RC[-1]-R[303]C[-1])^2"
Range("N10").Select
Selection.AutoFill Destination:=Range("N10:N309"), Type:= _
  xlFillDefault
Range("N10:N309").Select
Selection.Replace What:="-M*)^2", Replacement:="-M313)^2", LookAt _
  :=xlPart, SearchOrder:=xlByRows, MatchCase:=False
ActiveWindow.SmallScroll Down:=-2
ActiveWindow.SmallScroll ToRight:=3
ActiveWindow.LargeScroll ToRight:=-1
Range("B4").Select
ActiveCell.FormulaR1C1 = "=((6/(PI()^2))*1)"
Range("B5").Select
ActiveWindow.SmallScroll Down:=16
ActiveWindow.ScrollRow = 1
ActiveWindow.SmallScroll Down:=20
ActiveWindow.ScrollRow = 612
ActiveWindow.ScrollRow = 581
ActiveWindow.ScrollRow = 539
ActiveWindow.ScrollRow = 452
ActiveWindow.LargeScroll Down:=-9
ActiveWindow.ScrollRow = 339
ActiveWindow.ScrollRow = 309
Range("H310").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("H310").Select
Selection.Font.Bold = True
With Selection
  .HorizontalAlignment = xlCenter
  .VerticalAlignment = xlBottom
  .WrapText = False
  .Orientation = xlHorizontal
End With
Range("H311").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-301]C:R[-2]C)"
Range("H312").Select
ActiveWindow.ScrollRow = 1
ActiveWindow.ScrollRow = 465
ActiveWindow.ScrollRow = 287
ActiveWindow.ScrollRow = 322
ActiveWindow.LargeScroll ToRight:=1
ActiveWindow.ScrollRow = 203
ActiveWindow.ScrollRow = 269
ActiveWindow.ScrollRow = 3
ActiveWindow.SmallScroll ToRight:=-4
ActiveWindow.ScrollRow = 124
ActiveWindow.ScrollRow = 223
ActiveWindow.ScrollRow = 317
ActiveWindow.ScrollRow = 274
ActiveWindow.ScrollRow = 276
```

```
ActiveWindow.SmallScroll Down:=23
Range("H310").Select
Selection.Copy
Range("I310").Select
ActiveSheet.Paste
Range("I311").Select
Application.CutCopyMode = False
ActiveCell.FormulaR1C1 = "=SUM(R[-301]C:R[-2]C)"
Range("I312").Select
ActiveWindow.ScrollRow = 1
ActiveWindow.SmallScroll Down:=1
ActiveWindow.SmallScroll ToRight:=2
Range("O9").Select
ActiveCell.FormulaR1C1 = "(Ln Ti)^2"
Range("O9").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("O10").Select
ActiveCell.FormulaR1C1 = "=RC[-5]^2"
Range("O10").Select
Selection.AutoFill Destination:=Range("O10:O309"), Type:= _
    xlFillDefault
Range("O10:O309").Select
Range("O310").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("O310").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("O311").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-301]C:R[-2]C)"
Range("O312").Select
ActiveWindow.SmallScroll ToRight:=-7
ActiveWindow.LargeScroll Down:=-15
ActiveWindow.ScrollRow = 1
ActiveWindow.SmallScroll ToRight:=1
Range("B4").Select
ActiveCell.FormulaR1C1 = _
    "=((6/(PI()^2))*(R[307]C[13]-R[307]C[7]/300)/299)^(-0.5)"
Range("B4").Select
ActiveWindow.SmallScroll ToRight:=6
ActiveWindow.SmallScroll Down:=-1
```

```
ActiveCell.FormulaR1C1 = _
    "=((6/(PI()^2))*(R[307]C[13]-(R[307]C[7]^2)/300)/299)^(-0.5)"
Range("B4").Select
ActiveWindow.ScrollRow = 419
ActiveWindow.ScrollRow = 300
ActiveWindow.SmallScroll ToRight:=7
ActiveWindow.ScrollRow = 1
ActiveWindow.LargeScroll ToRight:=-1
ActiveCell.FormulaR1C1 = _
    "=((6/(PI()^2))*(R[307]C[13]-(R[307]C[7]^2)/300)/299)^(-0.5)"
Range("B4").Select
ActiveCell.FormulaR1C1 = _
    "=((6/(PI()^2))*(R[307]C[13]-(R[307]C[7]^2)/300)/299)^(-0.5)"
Range("I10").Select
ActiveWindow.SmallScroll ToRight:=2
Range("O10").Select
ActiveWindow.LargeScroll ToRight:=-1
Range("B4").Select
ActiveCell.FormulaR1C1 = _
    "=((6/(PI()^2))*(R[307]C[13]-(R[307]C[7]^2)/300)/299)^(-0.5)"
Range("B4").Select
ActiveWindow.SmallScroll ToRight:=2
Range("J10").Select
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=2
ActiveWindow.LargeScroll ToRight:=-1
Range("B4").Select
ActiveWindow.SmallScroll ToRight:=2
Range("J11").Select
ActiveWindow.LargeScroll ToRight:=-1
Range("B4").Select
ActiveCell.FormulaR1C1 = _
    "=((6/(PI()^2))*(R[307]C[13]-(R[307]C[7]^2)/300)/299)^(-0.5)"
Range("O10").Select
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=7
ActiveCell.FormulaR1C1 = "=RC[-6]^2"
Range("O10").Select
Selection.AutoFill Destination:=Range("O10:O309"), Type:= _
    xlFillDefault
Range("O10:O309").Select
ActiveWindow.LargeScroll Down:=-16
ActiveWindow.SmallScroll Down:=-12
ActiveWindow.LargeScroll ToRight:=-1
Range("B4").Select
ActiveWindow.SmallScroll ToRight:=3
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=13
Range("Q4").Select
ActiveCell.FormulaR1C1 = "=RC[-15]"
Range("Q5").Select
```

```
ActiveWindow.LargeScroll ToRight:=-2
With Toolbars(8)
    .Left = 398
    .Top = 139
End With
ActiveWindow.LargeScroll ToRight:=1
ActiveWindow.SmallScroll ToRight:=-9
ActiveWindow.SmallScroll Down:=3
Range("C6").Select
ActiveCell.FormulaR1C1 = _
    "=(300/RC[-1])+R[305]C[6]-(300*R[305]C[9]/R[305]C[8])"
Range("K310").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("K310").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("K311").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-301]C:R[-2]C)"
Range("K312").Select
ActiveWindow.LargeScroll Down:=-17
ActiveWindow.LargeScroll ToRight:=-1
Range("C6").Select
ActiveCell.FormulaR1C1 = _
    "=(300/RC[-1])+R[305]C[6]-(300*R[305]C[9]/R[305]C[8])"
Range("C6").Select
ActiveWindow.SmallScroll ToRight:=2
ActiveWindow.ScrollRow = 432
ActiveWindow.ScrollRow = 311
ActiveWindow.ScrollRow = 1
ActiveWindow.SmallScroll ToRight:=-2
Range("B10").Select
ActiveCell.FormulaR1C1 = "theta"
Range("B10").Select
Selection.Font.Bold = True
With Selection
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .WrapText = False
    .Orientation = xlHorizontal
End With
Range("B9").Select
ActiveCell.FormulaR1C1 = "=(300/R[302]C[9])^(1/R[-3]C)"
Range("B9").Select
ActiveCell.FormulaR1C1 = ""
Range("C9").Select
ActiveCell.FormulaR1C1 = ""
```

```
Range("B9").Select
ActiveCell.FormulaR1C1 = "=(300/R[302]C[9])^(1/R[-3]C)"
Range("B11").Select
ActiveCell.FormulaR1C1 = "=1/R[-2]C"
Range("B11").Select
ActiveWindow.SmallScroll ToRight:=9
Range("Q6").Select
With Toolbars(8)
  .Left = 58
  .Top = 25
End With
Range("P6").Select
ActiveCell.FormulaR1C1 = "((1/299)"
Range("S319").Select
ActiveWindow.ScrollRow = 302
Range("N310").Select
ActiveCell.FormulaR1C1 = "SUM"
Range("N310").Select
Selection.Font.Bold = True
With Selection
  .HorizontalAlignment = xlCenter
  .VerticalAlignment = xlBottom
  .WrapText = False
  .Orientation = xlHorizontal
End With
Range("N311").Select
ActiveCell.FormulaR1C1 = "=SUM(R[-301]C:R[-2]C)"
Range("N312").Select
ActiveWindow.LargeScroll Down:=-17
Range("P6").Select
ActiveCell.FormulaR1C1 = "(M311)/((N311)^(0.5))"
Range("P6").Select
ActiveWindow.LargeScroll ToRight:=0
ActiveCell.FormulaR1C1 = _
  "=R[305]C[-3])/(R[305]C[-2])^(0.5)"
Range("Q9").Select
ActiveCell.FormulaR1C1 = "=1/(((1/300)*R[302]C[-4])^(1/R[-3]C))"
Range("Q10").Select
ActiveWindow.LargeScroll ToRight:=-1
ActiveWindow.SmallScroll ToRight:=13
ActiveCell.FormulaR1C1 = "theta"
Range("Q10").Select
Selection.Font.Bold = True
With Selection
  .HorizontalAlignment = xlCenter
  .VerticalAlignment = xlBottom
  .WrapText = False
  .Orientation = xlHorizontal
End With
Range("Q11").Select
ActiveCell.FormulaR1C1 = "=1/R[-2]C"
```

```
   Range("Q12").Select
   ActiveWindow.LargeScroll ToRight:=-2
End Sub
```

APPENDIX D. PARAMETER CALCULATIONS FOR THE 20 AND 100 ENGINES

Row Names	V1	V2	V3
1	88.980852932	7.374101636	166.612433434
2	57.770945753	452.164077047	671.533220087
3	348.363054836	13.725217444	33.719418711
4	382.257870284	168.795667771	119.352260671
5	57.145359689	117.795729216	228.544288266
6	251.488400993	417.099715694	206.169150245
7	703.336492627	16.866142294	168.320579106
8	85.174113550	960.378668893	561.250692879
9	126.379607253	166.962891751	410.881188239
10	9.871149363	30.987527462	1171.995737937
11	184.016586935	55.183363300	170.715797827
12	19.085994032	206.177991742	124.656084035
13	171.708369596	7.411273439	458.366290751
14	957.877021060	334.941605556	252.553811069
15	88.483061291	129.532846338	55.052226186
16	97.662694658	64.764644477	521.223680534
17	213.546020387	8.955316669	314.364020817
18	420.340591055	182.036946622	345.506361499
19	46.823610226	411.857588415	109.450083602
20	25.300667331	361.149199434	744.757337329

	Column 1	*Column 2*	*Column 3*
Column 1	1		
Column 2	-0.099201099	1	
Column 3	-0.325682845	0.177625785	1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		MAXIMUM LIKELIHOOD ESTIMATOR			Number of observations		Failures times Ti										MODIFIED MOMENT METHOD	
2		shape parameter			n												shape parameter	
3		initial beta			60												initial beta	
4		1.001851															1.001851	
5		final beta															final beta	
6		0.973	-0.00065													7.811979	0.985725	
7		scale parameter															scale parameter	
8		landa															landa	
9		0.003973						Ti	Ln Ti	Ti^B4	Ti^B6	Ln TI * Ti^B6	Ti^Q6	(Ti^Q6-mean)^2	(Ln Ti)^2		0.00395	
10		theta		88.98085	7.374102	166.6124	88.98085	7.374102	1.997974	7.40142	6.986841	13.95953	7.166757	51413.43	3.9919		theta	
11		251.7053		57.77095	452.1641	671.5332	57.77095	7.411273	2.003002	7.438798	7.021108	14.06329	7.202367	51397.29	4.012018		253.1397	
12				348.3631	13.72522	33.71942	348.3631	8.955317	2.192247	8.991725	8.440626	18.50394	8.679408	50729.75	4.805949			
13				382.2579	168.7957	119.3523	382.2579	9.871149	2.289616	9.913067	9.279395	21.24625	9.553737	50336.66	5.242343			
14				57.14536	117.7957	228.5443	57.14536	13.72522	2.619235	13.79191	12.7881	33.49504	13.22152	48704.32	6.860391			
15				251.4884	417.0997	206.1692	251.4884	16.86614	2.825308	16.95456	15.62738	44.15217	16.19945	47398.78	7.982366			
16				703.3365	16.86614	168.3206	703.3365	19.08599	2.948955	19.19045	17.62525	51.97607	18.29923	46488.89	8.696334			
17				85.17411	960.3787	561.2507	85.17411	25.30067	3.230831	25.4524	23.18714	74.91374	24.16031	43995.8	10.43827			
18				126.3796	166.9629	410.8812	126.3796	30.98753	3.433585	31.18507	28.2439	96.97783	29.50533	41782.11	11.7895			
19				9.871149	30.98753	1171.996	9.871149	33.71942	3.518074	33.93968	30.66388	107.8778	32.06785	40741.09	12.37684			
20				184.0166	55.18336	170.7158	184.0166	46.82361	3.846388	47.15812	42.20483	162.3361	44.32199	35944.4	14.7947			
21				19.08599	206.178	124.6561	19.08599	55.05223	4.008282	55.46214	49.40532	198.0305	51.99069	33095.39	16.06633			
22				171.7084	7.411273	458.3663	171.7084	55.18336	4.010662	55.5945	49.51983	198.6073	52.11276	33050.99	16.08541			
23				957.877	334.9416	252.5538	957.877	57.14536	4.045598	57.57483	51.23211	207.2645	53.93868	32390.42	16.36686			
24				88.48306	129.5328	55.05223	88.48306	57.77095	4.056486	58.20629	51.77774	210.0357	54.52069	32181.27	16.45508			
25				97.66269	64.76464	521.2237	97.66269	64.76464	4.17076	65.2665	57.86709	241.3497	61.02129	29891.22	17.39524			
26				213.546	8.955317	314.364	213.546	85.17411	4.444698	85.87764	75.5421	335.7618	79.9379	23708.04	19.75534			
27				420.3406	182.0369	345.5064	420.3406	88.48306	4.482811	89.22022	78.39614	351.4351	82.99826	22774.98	20.0956			
28				46.82361	411.8576	109.4501	46.82361	88.98085	4.488421	89.72309	78.82524	353.8009	83.45851	22636.27	20.14592			
29				25.30067	361.1492	744.7573	25.30067	97.66269	4.58152	98.49432	86.299	395.3806	91.47988	20286.93	20.99032			
30							7.374102	109.4501	4.695469	110.4054	96.41775	452.7265	102.3544	17307.42	22.04743			
31							452.1641	117.7957	4.768952	118.84	103.564	493.8917	110.0435	15343.42	22.7429			
32							13.72522	119.3523	4.782079	120.4133	104.8953	501.6176	111.4767	14990.41	22.86828			
33							168.7957	124.6561	4.825559	125.7744	109.4281	528.0518	116.3583	13818.88	23.28602			
34							117.7957	126.3796	4.83929	127.5166	110.9	536.6772	117.944	13448.59	23.41873			
35							417.0997	129.5328	4.863934	130.7042	113.5914	552.5011	120.8442	12784.33	23.65786			
36							16.86614	166.6124	5.11567	168.1974	145.1179	742.3754	154.8791	6246.208	26.17008			
37							960.3787	166.9629	5.117772	168.5518	145.4149	744.2003	155.2002	6195.552	26.19159			
38							166.9629	168.3206	5.12587	169.925	146.5653	751.2749	156.4442	6001.273	26.27455			
39							30.98753	168.7957	5.128689	170.4055	146.9678	753.7523	156.8794	5934.025	26.30345			
40							55.18336	170.7158	5.14	172.3475	148.5943	763.7745	158.6384	5666.125	26.4196			
41							206.178	171.7084	5.145798	173.3515	149.4348	768.9614	159.5475	5530.082	26.47923			
42							7.411273	182.0369	5.20421	183.7987	158.1739	823.1704	169.0036	4213.102	27.0838			
43							334.9416	184.0166	5.215026	185.8013	159.8474	833.6083	170.8152	3981.217	27.1965			
44							129.5328	206.1692	5.328697	208.2125	178.5416	951.3939	191.0682	1835.595	28.39501			
45							64.76464	206.178	5.32874	208.2214	178.549	951.4413	191.0763	1834.903	28.39547			
46							8.955317	213.546	5.363852	215.6765	184.7545	990.9956	197.8055	1303.685	28.77091			
47							182.0369	228.5443	5.43173	230.8534	197.3685	1072.053	211.4932	502.6057	29.50369			
48							411.8576	251.4884	5.527397	254.0743	216.6226	1197.359	232.4079	2.262488	30.55212			
49							361.1492	252.5538	5.531624	255.1526	217.5154	1203.214	233.3784	0.284797	30.59887			
50							166.6124	314.364	5.750552	317.7276	269.1546	1547.788	289.5892	3099.941	33.06884			
51							671.5332	334.9416	5.813956	338.5651	286.2824	1664.433	308.2659	5528.496	33.80209			
52							33.71942	345.5064	5.845011	349.2642	295.0648	1724.657	317.8483	7045.298	34.16415			
53							119.3523	348.3631	5.853245	352.1573	297.4383	1740.979	320.4387	7486.855	34.26048			
54							228.5443	361.1492	5.889291	365.1071	308.0554	1814.228	332.029	9626.936	34.68375			
55							206.1692	382.2579	5.946095	386.4877	325.5611	1935.817	351.1508	13744.92	35.35605			
56							168.3206	410.8812	6.018304	415.4833	349.2574	2101.937	377.056	20490.18	36.21998			
57							561.2507	411.8576	6.020678	416.4725	350.0649	2107.628	377.9392	20743.81	36.24856			
58							410.8812	417.0997	6.033325	421.7832	354.3995	2138.207	382.6805	22132.05	36.40101			
59							1171.996	420.3406	6.041065	425.0666	357.0786	2157.135	385.6113	23012.67	36.49447			
60							170.7158	452.1641	6.114045	457.3096	383.3564	2343.858	414.3736	32566.36	37.38155			
61							124.6561	458.3663	6.127669	463.5941	388.4719	2380.427	419.9758	34619.7	37.54832			
62							458.3663	521.2237	6.256179	527.2938	440.2143	2754.06	476.6932	58942.7	39.13978			
63							252.5538	561.2507	6.330168	567.8647	473.0743	2994.639	512.7587	77755.46	40.07102			
64							55.05223	671.5332	6.509563	679.6725	563.2956	3666.808	611.9437	142908	42.37442			
65							521.2237	703.3365	6.555835	711.9222	589.2362	3862.935	640.5017	165315.1	42.97898			

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
66							314.364	744.7573	6.613058	753.9285	622.9742	4119.765	677.6683	196919.6	43.73254		
67							345.5064	957.877	6.864719	970.1243	795.8185	5463.071	868.4643	402656.5	47.12437		
68							109.4501	960.3787	6.867328	972.6626	797.8407	5479.034	870.7	405498.9	47.16019		
69							744.7573	1171.996	7.066463	1187.424	968.422	6843.319	1059.54	681662.2	49.9349		
70								SUM	SUM		SUM	SUM	SUM	SUM	SUM		
71								15284.8	296.1914		13008.26	77584.93	14034.72	3227644	1558.848		
72													mean				
73													233.9121				

Row Names	V1	V2	V3
1	450.634670353	134.620284266	763.602013650
2	395.264314931	300.696009190	201.433889226
3	467.906036969	301.510351673	22.166715862
4	200.004796884	261.419709923	234.019837022
5	62.187250449	351.566316602	293.954533661
6	188.420555520	530.441439163	446.499347177
7	517.045829232	212.650323103	4.372290612
8	104.406621033	91.440692508	286.231002289
9	185.580658025	189.658931128	461.452317203
10	278.805459607	142.687073420	63.785901768
11	242.374076871	140.003133652	57.571537504
12	191.638384699	35.982496826	135.685578893
13	115.651429115	807.515319374	56.217725189
14	318.738490247	220.511514630	96.958868093
15	389.324028552	121.699393963	391.967785281
16	108.549643630	833.642731237	587.010089214
17	647.283140422	270.006356536	600.593877204
18	135.277879814	235.260576261	294.610909130
19	167.565552866	19.382052477	80.687152199
20	620.417102151	43.940840941	348.339229735
21	549.652520297	147.263663645	402.329146256
22	160.858623029	198.697751449	131.936073250
23	269.748636292	216.407382425	272.564254146
24	119.882883842	42.753769287	568.105938757
25	290.367131217	46.018913125	470.135356798
26	111.526180647	59.040162467	442.572648090
27	315.869668084	112.068427109	799.073849390
28	258.473619251	740.012832046	730.391096774
29	936.074568617	525.344669063	53.458836253
30	38.275293391	264.758222487	109.033459770
31	200.399299969	130.448227050	564.453719467
32	16.387049764	689.920420726	124.595174314
33	282.346963237	302.451537677	17.105846707
34	298.005804703	792.704752916	277.592370478
35	488.070794225	600.835486551	77.330402620
36	651.631129729	186.384979015	302.942800237
37	488.356688854	282.631614706	12.094239822
38	251.322489919	639.047414057	315.553538531
39	102.245119335	121.065856465	248.600567386
40	120.292743494	281.418670027	6.512039507
41	394.628880256	329.862437764	42.172683323
42	275.823361976	400.377074953	205.279868273
43	89.822151642	568.897212277	126.945901458
44	413.319725633	280.829982419	74.621453179
45	172.347938766	138.081515648	169.690890193
46	295.648493372	49.777202258	271.146965769
47	427.883569557	120.358845277	86.140070778
48	106.453003164	336.696295308	392.945334315
49	439.747178352	145.848786206	357.670884257
50	95.424425759	158.394437081	233.214847248

Row Names	V1	V2	V3
51	78.318115166	233.292392917	150.556841342
52	400.322169035	703.176710554	131.985760999
53	284.082094482	700.398323845	54.502213913
54	673.813459742	628.345709323	27.810511050
55	416.157442911	309.802781935	61.701311275
56	502.727434228	208.984463635	7.000471997
57	395.249167889	181.615779362	246.169600960
58	532.055233005	282.774380172	51.530633142
59	256.526079351	265.073575146	366.480188363
60	791.013013348	483.679778572	383.122947523
61	116.674678096	203.429431816	501.860086251
62	216.019262031	50.783576381	346.268316228
63	652.055312871	297.710549841	19.143684755
64	57.143580125	444.863422850	53.260846841
65	305.430041853	64.522458661	344.328881540
66	134.875969833	153.848996778	74.573546508
67	207.795329813	197.474623424	36.572283433
68	55.079317773	148.585617029	153.485311966
69	332.356937878	183.167123218	2.771444009
70	33.292174799	281.412899135	521.055842681
71	329.237256924	624.659923834	410.145804201
72	134.999172015	924.566774196	959.801158180
73	310.283780658	196.949468983	61.420855422
74	145.927518733	98.091275956	105.358948583
75	96.240971550	448.577731504	68.349212862
76	504.457185330	3.995392340	54.819812986
77	779.185987694	63.103630524	510.654006981
78	64.295309451	227.558484641	681.717084473
79	132.919226467	80.090112060	305.523448635
80	225.006666940	354.405022103	115.791651829
81	322.833702315	15.741543588	273.716914308
82	552.256221658	238.880493230	103.236641646
83	234.905588128	267.774447132	384.884639091
84	241.821803683	27.660947307	35.559153707
85	360.121368725	54.400309193	74.757012406
86	109.375105806	169.046509653	306.549361255
87	211.877188712	54.427577933	47.413167726
88	328.701690509	233.519929404	411.204384691
89	271.788277670	132.777355320	359.728088427
90	54.715071977	52.466467632	253.216046834
91	157.314280817	49.880943701	746.100150202
92	93.608619218	110.699690363	573.390687287
93	218.283430910	434.282741182	109.443120744
94	1106.554817220	178.424617471	107.752314459
95	179.703059410	96.191152623	63.123472845
96	220.753490182	358.476353401	438.979104011
97	97.987549536	57.556717482	152.972644520
98	126.992796611	173.350825362	1004.747325381
99	656.574251587	250.152057881	312.989740534
100	189.330584986	142.057479837	63.629475350

	Column 1	*Column 2*	*Column 3*
Column 1	1		
Column 2	0.012345873	1	
Column 3	-0.151985402	0.027275574	1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		MAXIMUM LIKELIHOOD ESTIMATOR			Number of observations												MODIFIED MOMENT METHOD	
2		shape parameter			n			failures times Ti									shape parameter	
3		initial beta			300												initial beta	
4		1.26241															1.26241	
5		final beta															final beta	
6		1.261028	-1.9E-05													17.34948	1.288232	
7		scale parameter															scale parameter	
8		landa															landa	
9		0.003418						Ti	Ln Ti	Ti^B4	Ti^B6	Ln Ti * Ti^B6	Ti^Q6	(Ti^Q6-mean)^2	(Ln Ti)^2		0.003394	
10		theta		450.6347	134.6203	763.602	450.6347	2.771444	1.019368	3.621402	3.616305	3.686347	3.717989	2290781	1.039112		theta	
11		292.5569		395.2643	300.696	201.4339	395.2643	3.995392	1.385142	5.746651	5.735662	7.944705	5.955909	2284012	1.918618		294.6595	
12				467.906	301.5104	22.16672	467.906	4.372291	1.475287	6.439285	6.426172	9.480448	6.689317	2281796	2.176472			
13				200.0048	261.4197	234.0198	200.0048	6.51204	1.873653	10.64743	10.6199	19.898	11.17522	2268263	3.510574			
14				62.18725	351.5663	293.9545	62.18725	7.000472	1.945978	11.66534	11.63401	22.63953	12.26648	2264978	3.786829			
15				188.4206	530.4414	446.4993	188.4206	12.09424	2.492729	23.2626	23.18261	57.78798	24.80917	2227382	6.213699			
16				517.0458	212.6503	4.372291	517.0458	15.74154	2.756303	32.44627	32.32293	89.0918	34.83971	2197542	7.597208			
17				104.4066	91.44069	286.231	104.4066	16.38705	2.796491	34.13487	34.00322	95.08971	36.69092	2192057	7.820364			
18				185.5807	189.6589	461.4523	185.5807	17.10585	2.83942	36.03582	35.89471	101.9202	38.77717	2185884	8.062308			
19				278.8055	142.6871	63.7859	278.8055	19.14368	2.951973	41.53768	41.36859	122.1189	44.82767	2168030	8.714144			
20				242.3741	140.0031	57.57154	242.3741	19.38205	2.964348	42.19167	42.0192	124.5595	45.54801	2165909	8.787356			
21				191.6384	35.9825	135.6856	191.6384	22.16672	3.098592	49.98356	49.77001	154.2169	54.14712	2140672	9.601272			
22				115.6514	807.5153	56.21773	115.6514	27.66095	3.320022	66.104	65.80144	218.4622	72.02099	2088689	11.02254			
23				318.7385	220.5115	96.95887	318.7385	27.81051	3.325414	66.55554	66.25042	220.3101	72.52305	2087238	11.05838			
24				389.324	121.6994	391.9678	389.324	33.29217	3.505322	83.52575	83.12216	291.37	91.43863	2032940	12.28728			
25				108.5496	833.6427	587.0101	108.5496	35.55915	3.571198	90.76888	90.32207	322.558	99.53712	2009912	12.75345			
26				647.2831	270.0064	600.5939	647.2831	35.9825	3.583033	92.13521	91.68017	328.493	101.0663	2005578	12.83812			
27				135.2779	235.2606	294.6109	135.2779	36.57228	3.599291	94.04576	93.57919	336.8187	103.2054	1999524	12.95489			
28				167.5656	19.38205	80.68715	167.5656	38.27529	3.644805	99.60763	99.10723	361.2265	109.4375	1981938	13.2846			
29				620.4171	43.94084	348.3392	620.4171	42.17268	3.741773	112.5787	111.9981	419.0714	123.9987	1941151	14.00086			
30				549.6525	147.2637	402.3291	549.6525	42.75377	3.755457	114.5404	113.9476	427.9253	126.204	1935011	14.10346			
31				160.8586	198.6978	131.9361	160.8586	43.94084	3.782844	118.5697	117.9516	446.1925	130.7361	1922423	14.30991			
32				269.7486	216.4074	272.5643	269.7486	46.01891	3.829052	125.6921	125.0288	478.7419	138.7547	1900251	14.66164			
33				119.8829	42.75377	568.1059	119.8829	47.41317	3.8589	130.5185	129.8244	500.9794	144.1938	1885285	14.89111			
34				290.3671	46.01891	470.1354	290.3671	49.7772	3.907557	138.787	138.0396	539.3977	153.5213	1859758	15.269			
35				111.5262	59.04016	442.5726	111.5262	49.88094	3.909639	139.1522	138.4025	541.1038	153.9337	1858633	15.28528			
36				315.8697	112.0684	799.0738	315.8697	50.78358	3.927573	142.3386	141.5682	556.0193	157.5314	1848837	15.42583			
37				258.4736	740.0128	730.3911	258.4736	51.53063	3.942176	144.987	144.1994	568.4593	160.523	1840710	15.54076			
38				936.0746	525.3447	53.45884	936.0746	52.46647	3.960174	148.3189	147.5095	584.1633	164.2883	1830507	15.68298			
39				38.27529	264.7582	109.0335	38.27529	53.26085	3.975201	151.1594	150.3314	597.5976	167.4997	1821828	15.80223			
40				200.3993	130.4482	564.4537	200.3993	53.45884	3.978912	151.8691	151.0365	600.9608	168.3022	1819662	15.83174			
41				16.38705	689.9204	124.5952	16.38705	54.40031	3.99637	155.2534	154.3984	617.033	172.1302	1809349	15.97097			
42				282.347	302.4515	17.10585	282.347	54.42758	3.996871	155.3516	154.496	617.5005	172.2413	1809050	15.97498			
43				298.0058	792.7048	277.5924	298.0058	54.50221	3.998241	155.6206	154.7632	618.7806	172.5457	1808232	15.98593			
44				488.0708	600.8355	77.3304	488.0708	54.71507	4.002139	156.3882	155.5258	622.4358	173.4143	1805896	16.01712			
45				651.6311	186.385	302.9428	651.6311	54.81981	4.004052	156.7663	155.9013	624.2369	173.842	1804747	16.03243			
46				488.3567	282.6316	12.09424	488.3567	55.07932	4.008774	157.7037	156.8325	628.7062	174.9029	1801898	16.07027			
47				251.3225	639.0474	315.5535	251.3225	56.21773	4.029232	161.8296	160.9311	648.4288	179.5736	1789380	16.23471			
48				102.2451	121.0659	248.6006	102.2451	57.14358	4.045567	165.2014	164.2805	664.6077	183.3925	1779178	16.36661			
49				120.2927	281.4187	6.51204	120.2927	57.55672	4.052771	166.7106	165.7796	671.8669	185.1023	1774619	16.42495			
50				394.6289	329.8624	42.17268	394.6289	57.57154	4.053028	166.7648	165.8335	672.1277	185.1637	1774456	16.42704			
51				275.8234	400.3771	205.2799	275.8234	59.04016	4.078218	172.1531	171.1857	698.1326	191.2708	1758223	16.63186			
52				89.82215	568.8972	126.9459	89.82215	61.42086	4.117749	180.9624	179.9356	740.9299	201.2637	1731822	16.95586			
53				413.3197	280.83	74.62145	413.3197	61.70131	4.122305	182.0061	180.9723	746.0232	202.4484	1728705	16.9934			
54				172.3479	138.0815	169.6909	172.3479	62.18725	4.13015	183.8176	182.7715	754.8737	204.5047	1723302	17.05814			
55				295.6485	49.7772	271.147	295.6485	63.10363	4.144778	187.2436	186.1743	771.6513	208.3951	1713103	17.17919			
56				427.8836	120.3588	86.14007	427.8836	63.12347	4.145093	187.318	186.2481	772.0158	208.4795	1712882	17.18179			
57				106.453	336.6963	392.9453	106.453	63.62948	4.153077	189.2155	188.1328	781.33	210.6348	1707245	17.24805			
58				439.7472	145.8488	357.6709	439.7472	63.7859	4.155532	189.803	188.7162	784.2163	211.3021	1705502	17.26845			
59				95.42443	158.3944	233.2148	95.42443	64.29531	4.163487	191.7185	190.6187	793.6385	213.4785	1699822	17.33462			
60				78.31812	233.2924	150.5568	78.31812	64.52246	4.167013	192.574	191.4683	797.8511	214.4506	1697288	17.364			
61				400.3222	703.1767	131.9858	400.3222	68.34921	4.22463	207.103	205.8976	869.841	230.9735	1654509	17.8475			
62				284.0821	700.3983	54.50221	284.0821	74.57355	4.311786	231.1907	229.8173	990.923	258.4184	1584659	18.5915			
63				673.8135	628.3457	27.81051	673.8135	74.62145	4.312428	231.3782	230.0035	991.8735	258.6323	1584120	18.59704			
64				416.1574	309.8028	61.70131	416.1574	74.75701	4.314243	231.9089	230.5305	994.5647	259.2377	1582597	18.61269			

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
65				502.7274	208.9845	7.000472	502.7274	77.3304	4.348087	242.032	240.5822	1046.072	270.7903	1553664	18.90586			
66				395.2492	181.6158	246.1696	395.2492	78.31812	4.360779	245.9411	244.4636	1066.052	275.254	1542556	19.01639			
67				532.0552	282.7744	51.53063	532.0552	80.09011	4.383152	252.9866	251.459	1102.183	283.3029	1522627	19.21202			
68				256.5261	265.0736	366.4802	256.5261	80.68715	4.390579	255.3697	253.8251	1114.439	286.0265	1515913	19.27719			
69				791.013	483.6798	383.1229	791.013	86.14007	4.455975	277.3467	275.6442	1228.264	311.1667	1454639	19.85571			
70				116.6747	203.4294	501.8601	116.6747	89.82215	4.497832	292.3959	290.5843	1306.999	328.4058	1413352	20.23049			
71				216.0193	50.78358	346.2683	216.0193	91.44069	4.515691	299.063	297.2027	1342.075	336.0489	1395238	20.39146			
72				652.0553	297.7105	19.14368	652.0553	93.60862	4.539122	308.0416	306.1155	1389.496	346.3474	1371015	20.60363			
73				57.14358	444.8634	53.26085	57.14358	95.42443	4.558335	315.604	313.6224	1429.596	355.0263	1350766	20.77841			
74				305.43	64.52246	344.3289	305.43	96.19115	4.566337	318.8087	316.8034	1446.631	358.7054	1342227	20.85144			
75				134.876	153.849	74.57355	134.876	96.24097	4.566855	319.0171	317.0103	1447.74	358.9447	1341673	20.85617			
76				207.7953	197.4746	36.57228	207.7953	96.95887	4.574287	322.0242	319.9952	1463.75	362.3977	1333686	20.9241			
77				55.07932	148.5856	153.4853	55.07932	97.98755	4.58484	326.3432	324.2823	1486.782	367.3583	1322253	21.02076			
78				332.3569	183.1671	2.771444	332.3569	98.09128	4.585898	326.7794	324.7152	1489.111	367.8593	1321101	21.03046			
79				33.29217	281.4129	521.0558	33.29217	102.2451	4.627373	344.3447	342.15	1583.256	388.0482	1275098	21.41258			
80				329.2373	624.6599	410.1458	329.2373	103.2366	4.637024	348.5656	346.3393	1605.984	392.9027	1264159	21.50199			
81				134.9992	924.5668	959.8012	134.9992	104.4066	4.648293	353.5599	351.2963	1632.928	398.6482	1251272	21.60663			
82				310.2838	196.9495	61.42086	310.2838	105.3589	4.657373	357.636	355.3418	1654.959	403.3386	1240800	21.69112			
83				145.9275	98.09128	105.3589	145.9275	106.453	4.667704	362.3306	360.0011	1680.378	408.7422	1228791	21.78746			
84				96.24097	448.5777	68.34921	96.24097	107.7523	4.679835	367.9224	365.5509	1710.718	415.1803	1214559	21.90086			
85				504.4572	3.995392	54.81981	504.4572	108.5496	4.687208	371.3626	368.9652	1729.416	419.1422	1205842	21.96992			
86				779.186	63.10363	510.654	779.186	109.0335	4.691655	373.4534	371.0401	1740.792	421.5504	1200559	22.01162			
87				64.29531	227.5585	681.7171	64.29531	109.3751	4.694783	374.9312	372.5068	1748.839	423.2528	1196832	22.04099			
88				132.9192	80.09011	305.5234	132.9192	109.4431	4.695405	375.2256	372.799	1750.442	423.5919	1196090	22.04683			
89				225.0067	354.405	115.7917	225.0067	110.6997	4.706821	380.6724	378.2046	1780.141	429.8675	1182402	22.15416			
90				322.8337	15.74154	273.7169	322.8337	111.5262	4.714259	384.2638	381.7688	1799.757	434.0064	1173418	22.22424			
91				552.2562	238.8805	103.2366	552.2562	112.0684	4.71911	386.6239	384.111	1812.662	436.7267	1167532	22.27			
92				234.9056	267.7744	384.8846	234.9056	115.6514	4.750581	402.2935	399.6613	1898.623	454.7963	1128809	22.56802			
93				241.8218	27.66095	35.55915	241.8218	115.7917	4.751792	402.9093	400.2725	1902.012	455.5068	1127300	22.57953			
94				360.1214	54.40031	74.75701	360.1214	116.6747	4.75939	406.7921	404.1256	1923.391	459.9866	1117807	22.65179			
95				109.3751	169.0465	306.5494	109.3751	119.8829	4.786515	420.9635	418.1884	2001.665	476.3446	1083486	22.91073			
96				211.8772	54.42758	47.41317	211.8772	120.2927	4.789928	422.7811	419.9921	2011.732	478.4436	1079120	22.94341			
97				328.7017	233.5199	411.2044	328.7017	120.3588	4.790478	423.0745	420.2832	2013.357	478.7823	1078417	22.94868			
98				271.7883	132.7774	359.7281	271.7883	121.0659	4.796335	426.2142	423.3988	2030.762	482.4085	1070898	23.00483			
99				54.71507	52.46647	253.216	54.71507	121.6994	4.801554	429.0318	426.1947	2046.397	485.663	1064173	23.05492			
100				157.3143	49.88094	746.1002	157.3143	124.5952	4.82507	441.9593	439.0224	2118.314	500.6007	1033577	23.2813			
101				93.60862	110.6997	573.3907	93.60862	126.9459	4.843761	452.5117	449.4931	2177.237	512.8008	1008920	23.46202			
102				218.2834	434.2827	109.4431	218.2834	126.9928	4.84413	452.7227	449.7025	2178.418	513.0448	1008430	23.4656			
103				1106.555	178.4246	107.7523	1106.555	130.4482	4.870976	468.3288	465.1872	2265.916	531.0983	972496.6	23.72641			
104				179.7031	96.19115	63.12347	179.7031	131.9361	4.882318	475.0822	471.8879	2303.906	538.9146	957141.6	23.83702			
105				220.7535	358.4764	438.9791	220.7535	131.9858	4.882694	475.308	472.112	2305.178	539.1761	956630.1	23.8407			
106				97.98755	57.55672	152.9726	97.98755	132.7774	4.888674	478.9096	475.6854	2325.471	543.3455	948491.5	23.89913			
107				126.9928	173.3508	1004.747	126.9928	132.9192	4.889742	479.5557	476.3264	2329.113	544.0935	947035.1	23.90957			
108				656.5743	250.1521	312.9897	656.5743	134.6203	4.902458	487.3163	484.0263	2372.919	553.0801	929625	24.0341			
109				189.3306	142.0575	63.62948	189.3306	134.876	4.904356	488.4851	485.1859	2379.524	554.4337	927016.6	24.0527			
110							134.6203	134.9992	4.905269	489.0484	485.7448	2382.709	555.0862	925760.5	24.06166			
111							300.696	135.2779	4.907331	490.3233	487.0097	2389.918	556.563	922921	24.0819			
112							301.5104	135.6856	4.91034	492.1896	488.8613	2400.476	558.7247	918772.1	24.11144			
113							261.4197	138.0815	4.927844	503.1866	499.7719	2462.798	571.4666	894507.7	24.28365			
114							351.5663	140.0031	4.941665	512.0429	508.5584	2513.125	581.7322	875195	24.42005			
115							530.4414	142.0575	4.956232	521.5462	517.9866	2567.262	592.7518	854698.3	24.56423			
116							212.6503	142.6871	4.960654	524.4659	520.8832	2583.921	596.1382	848448.3	24.60809			
117							91.44069	145.8488	4.98257	539.1792	535.4798	2668.066	613.2091	817291.2	24.82601			
118							189.6589	145.9275	4.98311	539.5467	535.8443	2670.171	613.6356	816520.3	24.83139			
119							142.6871	147.2637	4.992225	545.7907	542.0387	2705.979	620.8832	803474.8	24.92231			
120							140.0031	148.5856	5.001161	551.9831	548.1817	2741.545	628.0724	790638.1	25.01161			
121							35.9825	150.5568	5.014341	561.2437	557.3684	2794.835	638.8269	771628.4	25.14361			
122							807.5153	152.9726	5.030259	572.6363	568.6698	2860.556	652.0623	748551	25.30351			
123							220.5115	153.4853	5.033605	575.06	571.0741	2874.562	654.8789	743685.2	25.33718			
124							121.6994	153.849	5.035972	576.7808	572.781	2884.509	656.8786	740240.3	25.36101			
125							833.6427	157.3143	5.058246	593.2294	589.0975	2979.8	676.0001	707702.6	25.58585			
126							270.0064	158.3944	5.065088	598.3761	594.2028	3009.689	681.9855	697668.1	25.65512			
127							235.2606	160.8586	5.080526	610.1519	605.8835	3078.207	695.6839	674972.1	25.81174			
128							19.38205	167.5656	5.121375	642.4416	637.9113	3266.983	733.273	614621.1	26.22848			
129							43.94084	169.0465	5.130174	649.6178	645.029	3309.111	741.6323	601584	26.31868			
130							147.2637	169.6909	5.133978	652.7454	648.1311	3327.491	745.2761	595944.8	26.35774			
131							198.6978	172.3479	5.149515	665.6746	660.9548	3403.597	760.3432	572909.1	26.51751			
132							216.4074	173.3508	5.155317	670.5684	665.8085	3432.454	766.0476	564306.2	26.5773			
133							42.75377	178.4246	5.184166	695.4399	690.476	3579.542	795.0525	521570.2	26.87558			

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
134							46.01891	179.7031	5.191306	701.7363	696.7206	3616.89	802.3988	511013.4	26.94966			
135							59.04016	181.6158	5.201893	711.1786	706.085	3672.979	813.4178	495380.8	27.05969			
136							112.0684	183.1671	5.210399	718.8561	713.6991	3718.657	822.3796	482845.9	27.14826			
137							740.0128	185.5807	5.22349	730.8344	725.5784	3790.051	836.3656	463604.6	27.28484			
138							525.3447	186.385	5.227814	734.8353	729.5462	3813.932	841.0382	457263.4	27.33004			
139							264.7582	188.4206	5.238676	744.9812	739.6079	3874.566	852.8896	441375.8	27.44373			
140							130.4482	189.3306	5.243495	749.5263	744.1153	3901.764	858.1998	434348.1	27.49424			
141							689.9204	189.6589	5.245227	751.1676	745.743	3911.592	860.1176	431824	27.51241			
142							302.4515	191.6384	5.25561	761.0783	755.5712	3970.988	871.6994	416736.6	27.62144			
143							792.7048	196.9495	5.282947	787.802	782.072	4131.645	902.9444	377372.2	27.90953			
144							600.8355	197.4746	5.28561	790.4548	784.7026	4147.632	906.0472	373569.7	27.93767			
145							186.385	198.6978	5.291785	796.6405	790.8366	4184.937	913.2831	364776.9	28.00299			
146							282.6316	200.0048	5.298341	803.2617	797.4023	4224.909	921.0297	355479.5	28.07242			
147							639.0474	200.3993	5.300312	805.2623	799.3862	4236.996	923.3707	352693.5	28.09331			
148							121.0659	201.4339	5.305461	810.5141	804.5939	4268.742	929.5163	345431.8	28.14792			
149							281.4187	203.4294	5.315319	820.6638	814.6583	4330.169	941.3958	331608.9	28.25262			
150							329.8624	205.2799	5.324374	830.0988	824.014	4387.359	952.4415	319009.5	28.34896			
151							400.3771	207.7953	5.336554	842.9605	836.7673	4465.454	967.503	302222.6	28.4788			
152							568.8972	208.9845	5.34226	849.0548	842.8102	4502.511	974.6414	294425	28.53974			
153							280.83	211.8772	5.356007	863.9181	857.5479	4593.032	992.0552	275830.4	28.68681			
154							138.0815	212.6503	5.359649	867.8996	861.4958	4617.315	996.721	270951.2	28.72584			
155							49.7772	216.0193	5.375368	885.2935	878.7422	4723.562	1017.109	250141.4	28.89458			
156							120.3588	216.4074	5.377163	887.3019	880.7336	4735.848	1019.464	247791.5	28.91388			
157							336.6963	218.2834	5.385794	897.0235	890.3726	4795.364	1030.863	236572.6	29.00678			
158							145.8488	220.5115	5.39595	908.5978	901.8484	4866.329	1044.439	223551.4	29.11628			
159							158.3944	220.7535	5.397047	909.8567	903.0965	4874.054	1045.915	222157.2	29.12811			
160							233.2924	225.0067	5.41613	932.0423	925.0929	5010.423	1071.946	198295.9	29.33446			
161							703.1767	227.5585	5.427407	945.4062	938.3426	5092.767	1087.633	184571.3	29.45675			
162							700.3983	233.2148	5.45196	975.1686	967.8498	5276.679	1122.584	155761.5	29.72387			
163							628.3457	233.2924	5.452293	975.578	968.2556	5279.213	1123.065	155382.1	29.72749			
164							309.8028	233.5199	5.453267	976.7793	969.4467	5286.652	1124.477	154271.5	29.73813			
165							208.9845	234.0198	5.455406	979.4198	972.0645	5303.006	1127.579	151844.4	29.76145			
166							181.6158	234.9056	5.459184	984.102	976.7063	5332.019	1133.08	147587.5	29.80269			
167							282.7744	235.2606	5.460694	985.9798	978.568	5343.66	1135.286	145897.1	29.81918			
168							265.0736	238.8805	5.475963	1005.17	997.5934	5462.785	1157.839	129176.8	29.98618			
169							483.6798	241.8218	5.488201	1020.82	1013.108	5560.139	1176.237	116290.4	30.12035			
170							203.4294	242.3741	5.490482	1023.764	1016.026	5578.474	1179.699	113941.4	30.1454			
171							50.78358	246.1696	5.506021	1044.044	1036.131	5704.959	1203.551	98407.65	30.31626			
172							297.7105	248.6006	5.515847	1057.077	1049.05	5786.402	1218.884	89023	30.42457			
173							444.8634	250.1521	5.522069	1065.412	1057.313	5838.556	1228.692	83266.26	30.49325			
174							64.52246	251.3225	5.526737	1071.709	1063.555	5877.99	1236.103	79044.21	30.54482			
175							153.849	253.216	5.534243	1081.912	1073.67	5941.951	1248.113	72434.93	30.62785			
176							197.4746	256.5261	5.54723	1099.797	1091.399	6054.24	1269.171	61543.7	30.77176			
177							148.5856	258.4736	5.554794	1110.348	1101.858	6120.592	1281.597	55532.66	30.85573			
178							183.1671	261.4197	5.566127	1126.348	1117.719	6221.363	1300.446	47004.31	30.98177			
179							281.4129	264.7582	5.578817	1144.537	1135.748	6336.133	1321.88	38169.86	31.1232			
180							624.6599	265.0736	5.580007	1146.259	1137.455	6347.005	1323.908	37381.29	31.13648			
181							924.5668	267.7744	5.590145	1161.023	1152.089	6440.344	1341.311	30954.66	31.24972			
182							196.9495	269.7486	5.597491	1171.839	1162.81	6508.819	1354.064	26629.88	31.3319			
183							98.09128	270.0064	5.598446	1173.252	1164.211	6517.774	1355.731	26088.66	31.34259			
184							448.5777	271.147	5.602661	1179.513	1170.417	6557.447	1363.113	23758.36	31.38981			
185							3.995392	271.7883	5.605023	1183.036	1173.908	6579.784	1367.268	22494.84	31.41629			
186							63.10363	272.5643	5.607874	1187.301	1178.137	6606.842	1372.299	21011.06	31.44826			
187							227.5585	273.7169	5.612094	1193.643	1184.423	6647.092	1379.779	18898.35	31.4956			
188							80.09011	275.8234	5.619761	1205.252	1195.929	6720.832	1393.474	15320.79	31.58171			
189							354.405	277.5924	5.626154	1215.018	1205.609	6782.941	1404.997	12600.84	31.65361			
190							15.74154	278.8055	5.630514	1221.725	1212.256	6825.627	1412.912	10886.6	31.70269			
191							238.8805	280.83	5.637749	1232.935	1223.367	6897.039	1426.143	8300.694	31.78422			
192							267.7744	281.4129	5.639823	1236.167	1226.57	6917.64	1429.957	7620.162	31.8076			
193							27.66095	281.4187	5.639843	1236.199	1226.602	6917.844	1429.995	7613.568	31.80783			
194							54.40031	282.347	5.643137	1241.349	1231.707	6950.689	1436.074	6589.587	31.84499			
195							169.0465	282.6316	5.644144	1242.929	1233.273	6960.769	1437.94	6290.22	31.85637			
196							54.42758	282.7744	5.644649	1243.721	1234.058	6965.827	1438.876	6142.662	31.86207			
197							233.5199	284.0821	5.649263	1250.987	1241.259	7012.201	1447.453	4871.659	31.91418			
198							132.7774	286.231	5.656799	1262.945	1253.111	7088.599	1461.574	3099.922	31.99938			
199							52.46647	290.3671	5.671146	1286.027	1275.989	7236.319	1488.838	807.2844	32.1619			
200							49.88094	293.9545	5.683425	1306.117	1295.9	7365.152	1512.576	21.85311	32.30132			
201							110.6997	294.6109	5.685656	1309.8	1299.55	7388.795	1516.928	0.103934	32.32668			
202							434.2827	295.6485	5.689171	1315.626	1305.324	7426.214	1523.814	43.07825	32.36667			

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
203							178.4246	297.7105	5.696122	1327.221	1316.816	7500.742	1537.519	410.8177	32.4458			
204							96.19115	298.0058	5.697113	1328.883	1318.463	7511.43	1539.484	494.318	32.4571			
205							358.4764	300.696	5.7061	1344.045	1333.489	7609.023	1557.41	1612.799	32.55958			
206							57.55672	301.5104	5.708804	1348.642	1338.045	7638.637	1562.846	2078.927	32.59045			
207							173.3508	302.4515	5.711921	1353.958	1343.314	7672.904	1569.134	2691.821	32.62604			
208							250.1521	302.9428	5.713544	1356.735	1346.066	7690.808	1572.418	3043.38	32.64459			
209							142.0575	305.43	5.721721	1370.813	1360.017	7781.64	1589.068	5157.755	32.73809			
210							763.602	305.5234	5.722027	1371.342	1360.542	7785.057	1589.694	5248.072	32.74159			
211							201.4339	306.5494	5.725379	1377.158	1366.305	7822.616	1596.574	6292.218	32.77996			
212							22.16672	309.8028	5.735936	1395.634	1384.616	7942.071	1618.436	10238.46	32.90096			
213							234.0198	310.2838	5.737487	1398.37	1387.328	7959.776	1621.674	10904.17	32.91876			
214							293.9545	312.9897	5.74617	1413.783	1402.602	8059.591	1639.915	15046.62	33.01847			
215							446.4993	315.5535	5.754328	1428.419	1417.106	8154.492	1657.241	19597.19	33.11229			
216							4.372291	315.8697	5.75533	1430.225	1418.896	8166.216	1659.38	20200.68	33.12382			
217							286.231	318.7385	5.764371	1446.643	1435.166	8272.83	1678.82	26104.66	33.22797			
218							461.4523	322.8337	5.777137	1470.147	1458.457	8425.709	1706.658	35875.22	33.37532			
219							63.7859	328.7017	5.795151	1503.961	1491.966	8646.167	1746.725	52658.38	33.58377			
220							57.57154	329.2373	5.796779	1507.055	1495.032	8666.369	1750.392	54354.87	33.60264			
221							135.6856	329.8624	5.798676	1510.669	1498.613	8689.97	1754.675	56370.29	33.62464			
222							56.21773	332.3569	5.80621	1525.105	1512.918	8784.318	1771.788	64788.98	33.71207			
223							96.95887	336.6963	5.819181	1550.285	1537.87	8949.142	1801.644	80879.6	33.86287			
224							391.9678	344.3289	5.841597	1594.782	1581.961	9241.178	1854.429	113688.8	34.12426			
225							587.0101	346.2683	5.847214	1606.13	1593.205	9315.813	1867.895	122951.5	34.18991			
226							600.5939	348.3392	5.853177	1618.266	1605.23	9395.697	1882.299	133260	34.25968			
227							294.6109	351.5663	5.862398	1637.215	1624.006	9520.57	1904.793	150188.8	34.36771			
228							80.68715	354.405	5.87044	1653.921	1640.559	9630.805	1924.629	165957	34.46207			
229							348.3392	357.6709	5.879613	1673.185	1659.646	9758.077	1947.507	185120.3	34.56985			
230							402.3291	358.4764	5.881863	1677.943	1664.361	9789.54	1953.158	190015.6	34.59631			
231							131.9361	359.7281	5.885348	1685.343	1671.693	9838.493	1961.949	197756.3	34.63733			
232							272.5643	360.1214	5.886441	1687.669	1673.998	9853.888	1964.712	200221.9	34.65019			
233							568.1059	366.4802	5.903944	1725.376	1711.357	10103.76	2009.517	242325.7	34.85656			
234							470.1354	383.1229	5.948356	1824.873	1809.935	10766.14	2127.838	372817	35.38294			
235							442.5726	384.8846	5.952944	1835.472	1820.436	10836.95	2140.451	388378.5	35.43754			
236							799.0738	389.324	5.964412	1862.239	1846.954	11016	2172.308	429100.6	35.57421			
237							730.3911	391.9678	5.97118	1878.218	1862.784	11123.02	2191.33	454383.1	35.65499			
238							53.45884	392.9453	5.973671	1884.133	1868.644	11162.67	2198.373	463927.5	35.68474			
239							109.0335	394.6289	5.977946	1894.329	1878.746	11231.04	2210.514	480614	35.73584			
240							564.4537	395.2492	5.979516	1898.089	1882.471	11256.26	2214.991	486841.6	35.75462			
241							124.5952	395.2643	5.979555	1898.181	1882.562	11256.88	2215.1	486994.2	35.75507			
242							17.10585	400.3222	5.99227	1928.895	1912.99	11463.15	2251.682	539389.5	35.9073			
243							277.5924	400.3771	5.992407	1929.229	1913.321	11465.4	2252.08	539974	35.90894			
244							77.3304	402.3291	5.997271	1941.111	1925.092	11545.3	2266.235	560977.3	35.96725			
245							302.9428	410.1458	6.016513	1988.841	1972.375	11866.82	2323.113	649414.5	36.19843			
246							12.09424	411.2044	6.01909	1995.324	1978.797	11910.56	2330.84	661928.1	36.22945			
247							315.5535	413.3197	6.024221	2008.29	1991.642	11998.09	2346.298	687319.8	36.29124			
248							248.6006	416.1574	6.031064	2025.712	2008.901	12115.81	2367.071	722194.1	36.37373			
249							6.51204	427.8836	6.058851	2098.034	2080.542	12605.69	2453.339	876261.1	36.70968			
250							42.17268	434.2827	6.073696	2137.722	2119.856	12875.36	2500.707	967185.3	36.88978			
251							205.2799	438.9791	6.084452	2166.947	2148.805	13074.3	2535.598	1037031	37.02055			
252							126.9459	439.7472	6.0862	2171.734	2153.547	13106.92	2541.315	1048707	37.04183			
253							74.62145	442.5726	6.092605	2189.365	2171.01	13227.11	2562.369	1092272	37.11983			
254							169.6909	444.8634	6.097767	2203.68	2185.19	13324.78	2579.468	1128305	37.18277			
255							271.147	446.4993	6.101438	2213.915	2195.328	13394.66	2591.694	1154428	37.22754			
256							86.14007	448.5777	6.106082	2226.933	2208.223	13483.59	2607.245	1188088	37.28424			
257							392.9453	450.6347	6.110657	2239.832	2220.999	13571.76	2622.657	1221923	37.34013			
258							357.6709	461.4523	6.134379	2307.922	2288.441	14038.17	2704.04	1408469	37.6306			
259							233.2148	467.906	6.148268	2348.744	2328.874	14318.54	2752.856	1526721	37.80119			
260							150.5568	470.1354	6.153021	2362.88	2342.875	14415.76	2769.764	1568789	37.85966			
261							131.9858	483.6798	6.181423	2449.139	2428.309	15010.41	2872.983	1838011	38.20999			
262							54.50221	488.0708	6.19046	2477.241	2456.141	15204.65	2906.627	1930366	38.3218			
263							27.81051	488.3567	6.191046	2479.073	2457.956	15217.32	2908.82	1936466	38.32905			
264							61.70131	501.8601	6.218321	2565.921	2543.968	15819.21	3012.844	2236800	38.66752			
265							7.000472	502.7274	6.220048	2571.52	2549.513	15858.1	3019.554	2256914	38.689			
266							246.1696	504.4572	6.223483	2582.695	2560.58	15935.73	3032.944	2297328	38.73174			
267							51.53063	510.654	6.235692	2622.811	2600.309	16214.73	3081.025	2445390	38.88386			
268							366.4802	517.0458	6.248132	2664.323	2641.419	16503.94	3130.795	2603525	39.03915			
269							383.1229	521.0558	6.255857	2690.436	2667.279	16686.12	3162.11	2705561	39.13575			
270							501.8601	525.3447	6.264055	2718.422	2694.994	16881.59	3195.679	2817121	39.23838			
271							346.2683	530.4414	6.27371	2751.758	2728.006	17114.72	3235.675	2952980	39.35943			

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
272							19.14368	532.0552	6.276747	2762.331	2738.477	17188.73	3248.362	2996745	39.39756			
273							53.26085	549.6525	6.309286	2878.164	2853.181	18001.53	3387.42	3497532	39.80709			
274							344.3289	552.2562	6.314012	2895.386	2870.234	18122.7	3408.105	3575330	39.86675			
275							74.57355	564.4537	6.335858	2976.349	2950.405	18693.35	3505.382	3952666	40.1431			
276							36.57228	568.1059	6.342308	3000.681	2974.499	18865.19	3534.628	4069810	40.22487			
277							153.4853	568.8972	6.3437	3005.958	2979.724	18902.48	3540.971	4095445	40.24253			
278							2.771444	573.3907	6.351567	3035.962	3009.434	19114.62	3577.042	4242741	40.34241			
279							521.0558	587.0101	6.375042	3127.279	3099.851	19761.68	3686.867	4707236	40.64116			
280							410.1458	600.5939	6.397919	3218.912	3190.58	20413.07	3797.139	5197891	40.93337			
281							959.8012	600.8355	6.398321	3220.546	3192.198	20424.71	3799.107	5206868	40.93851			
282							61.42086	620.4171	6.430392	3353.611	3323.944	21374.26	3959.353	5963865	41.34994			
283							105.3589	624.6599	6.437207	3382.589	3352.635	21581.6	3994.269	6135618	41.43764			
284							68.34921	628.3457	6.443091	3407.805	3377.6	21762.18	4024.656	6287079	41.51342			
285							54.81981	639.0474	6.459979	3481.238	3450.302	22288.88	4113.175	6738822	41.73132			
286							510.654	647.2831	6.472784	3537.971	3506.468	22696.61	4181.589	7098696	41.89693			
287							681.7171	651.6311	6.479479	3568	3536.197	22912.71	4217.809	7293013	41.98364			
288							305.5234	652.0553	6.480129	3570.932	3539.1	22933.82	4221.346	7312131	41.99208			
289							115.7917	656.5743	6.487036	3602.202	3570.057	23159.09	4259.071	7517579	42.08163			
290							273.7169	673.8135	6.512953	3722.01	3688.664	24024.09	4403.672	8331430	42.41856			
291							103.2366	681.7171	6.524615	3777.209	3743.308	24423.64	4470.327	8720657	42.5706			
292							384.8846	689.9204	6.536576	3834.679	3800.199	24840.29	4539.744	9135467	42.72683			
293							35.55915	700.3983	6.551649	3908.345	3873.122	25375.34	4628.756	9681465	42.92411			
294							74.75701	703.1767	6.555608	3927.927	3892.507	25517.75	4652.424	9829308	42.976			
295							306.5494	730.3911	6.59358	4120.803	4083.429	26924.41	4885.663	11346198	43.4753			
296							47.41317	740.0128	6.606668	4189.451	4151.379	27426.78	4968.731	11912716	43.64806			
297							411.2044	746.1002	6.61486	4233.003	4194.488	27745.95	5021.447	12279390	43.75637			
298							359.7281	763.602	6.638047	4358.74	4318.943	28669.34	5173.7	13369624	44.06366			
299							253.216	779.186	6.65825	4471.337	4430.388	29498.63	5310.12	14385855	44.33229			
300							746.1002	791.013	6.673314	4557.186	4515.357	30132.39	5414.178	15186045	44.53313			
301							573.3907	792.7048	6.675451	4569.494	4527.538	30223.36	5429.1	15302563	44.56164			
302							109.4431	799.0738	6.683453	4615.891	4573.458	30566.5	5485.359	15745879	44.66855			
303							107.7523	807.5153	6.693962	4677.534	4634.468	31022.95	5560.122	16344808	44.80913			
304							63.12347	833.6427	6.725805	4869.396	4824.351	32447.64	5792.947	18281576	45.23645			
305							438.9791	924.5668	6.829325	5549.207	5497.087	37541.39	6619.364	26031560	46.63968			
306							152.9726	936.0746	6.841695	5636.543	5583.507	38200.65	6725.69	27127842	46.80879			
307							1004.747	959.8012	6.866726	5817.498	5762.56	39569.92	6946.1	29472399	47.15193			
308							312.9897	1004.747	6.912491	6163.5	6104.908	42200.13	7367.928	34230423	47.78254			
309							63.62948	1106.555	7.009007	6962.139	6895.036	48327.35	8343.399	46596295	49.12617			
310								SUM	SUM		SUM	SUM	SUM	SUM	SUM			
311								81587.27	1565.838		386440.6	2323460	455175.2	6.88E+08	8481.441			
312													mean					
313													1517.251					

APPENDIX E. FUNCTION "*tocompare*"

```
function(v, betamle, thetamle, betammm, thetammm)
{ #   This function calculates the kolmogorov-smirnov goodness of the fit,
  #   and plots of the empical distribution and theoretical distribution
  #   for a set of data and two sets of parameters of weibull distribution.
win.graph()
a <- ks.gof(v, dist = "weibull", shape = betamle, scale = thetamle)
b <- ks.gof(v, dist = "weibull", shape = betammm, scale = thetammm)
par(mfrow = c(1, 2))
cdf.compare(v, dist = "weibull", shape = betamle, scale = thetamle)
cdf.compare(v, dist = "weibull", shape = betammm, scale = thetammm)
list(a, b)
}
```

```
tocompare(a20.1,0.973,251.71,0.9857,253.14)
[[1]]:

One-sample Kolmogorov-Smirnov Test; hypothesized distribution = weibull

data: v
ks = 0.0546, p-value = 0.994
alternative hypothesis: True cdf does not equal the weibull Distn. for at least one sample point.

[[2]]:

One-sample Kolmogorov-Smirnov Test; hypothesized distribution = weibull
data: v
ks = 0.0509, p-value = 0.9977
Alternative hypothesis: True cdf does not equal the weibull Distn. for at least one sample point.
```

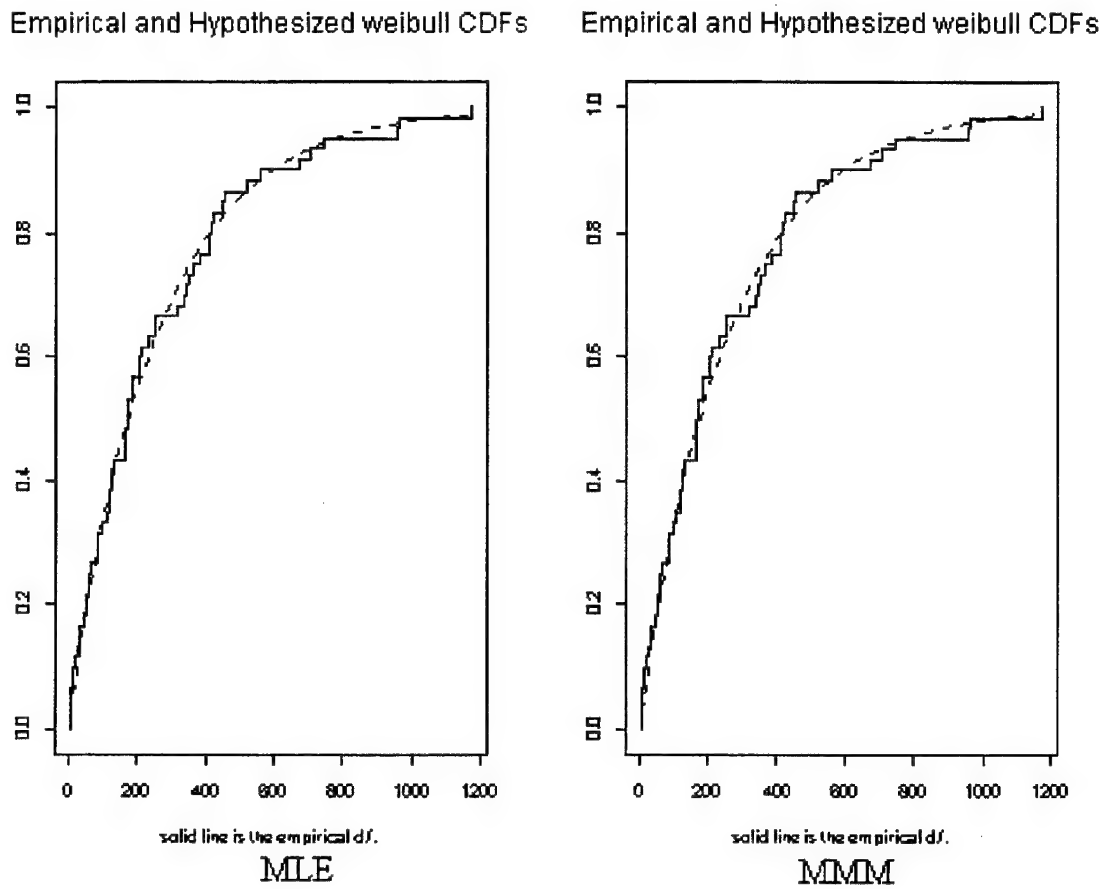

Figure 5. Kolmogorov-Smirnov Test; Hypothesized Distribution = Weibull. (a) ks = 0.0546, p-value = 0.994, (b) ks = 0.0509, p-value = 0.9977

tocompare(a100.1,1.261,292.56,1.2882,294.66)

[[1]]:

One-sample Kolmogorov-Smirnov Test; hypothesized distribution = weibull

data: v

ks = 0.0319, p-value = 0.9207

Alternative hypothesis: True cdf does not equal the weibull Distn. for at least one sample point.

[[2]]:

One-sample Kolmogorov-Smirnov Test; hypothesized distribution = weibull

data: v

ks = 0.0382, p-value = 0.7741

Alternative hypothesis: True cdf does not equal the weibull Distn. for at least one sample point.

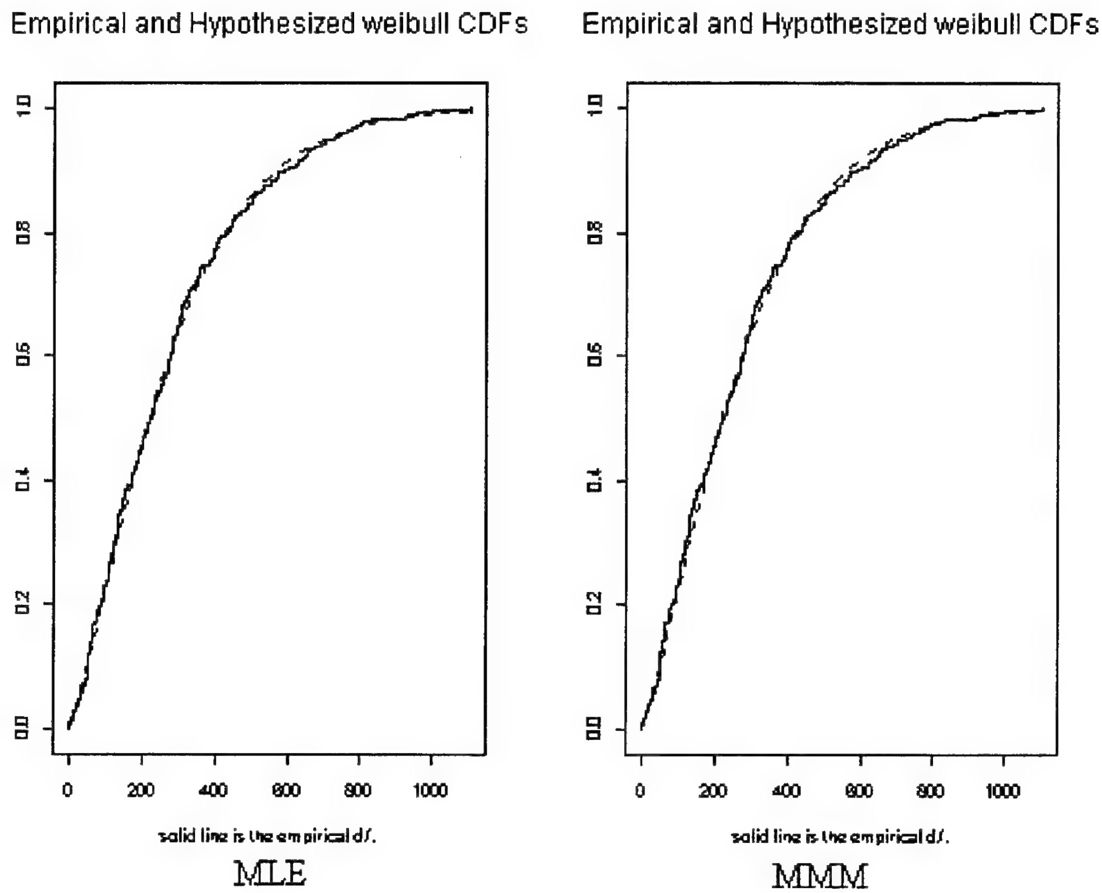

Figure 6. Kolmogorov-Smirnov Test; Hypothesized Distribution = Weibull. (a) ks = 0.0319, p-value = 0.9207, (b) ks = 0.0382, p-value = 0.7741

LIST OF REFERENCES

1. Caudill, Michel R., "Methods For Performance Goal Setting Of Fielded Jet Engines," Master's Thesis, Naval Postgraduate School, Monterey, California, June 1995.

2. Malsbury, John A., "Methods For Determining Goals And Expectations For Fielded Jet Engines" Master's Thesis, Naval Postgraduate School, Monterey, California, December 1995.

3. Mlikan, Mark E., "Methods For Determining Performance Expectations And Optimal No Build Times of Fielded Jet Engines" Master's Thesis, Naval Postgraduate School, Monterey, California, June 1996.

4. Thoman, Darrel R., Lee J. Bain, and Charles E. Antle, "Inferences on the Parameters of the Weibull Distribution," *Technometrics*, Vol. 11, No. 3, August 1969, p. 445-460.

5. Youngreen, "Reliability," Lecture, Naval Post graduate School, Monterey, California, Winter , 1997.

6. Law, Averill M., and W. David Kelton, *Simulation Modeling and Analysis*, Second Edition, New York: McGraw-Hill, 1991.

BIBLIOGRAPHY

Gibbons, Jean Dickinson, *Nonparametric Methods for Quantitative Analysis*, Second Edition, Columbus, Ohio: American Sciences Press, 1985.

Kececioglu, Dimitri, *Reliability and Life Testing Handbook*, Vol. I, Englewood Cliffs, New Jersey: Prentice-Hall, 1993.

Leemis, Lawrence M., *Reliability: Probabilistic Models and Statistical Methods*, Englewood Cliffs, New Jersey: Prentice-Hall, 1995.

Nelson, Wayne, *Applied Life Data Analysis*, New York: John Wiley & Sons, 1982.

Statistical Sciences, S-PLUS Guide to Statistical and Mathematical Analysis, Version 3.3, Seattle: StatSci, 1995.

User's Guide - Microsoft Excel, Redmond, Washington: Microsoft Corporation, 1994.

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